KOREA INSTITUTE FOR ADVANCED STUDY

ENHANCING QUANTUM STATE DISCRIMINATION WITH INDEFINITE CAUSAL ORDER

Quantum Resources 2025,

Jeju, March 21 2025

OVERVIEW

- Motivation
- Quantum State Discrimination
- Indefinite Causal Order
- Results
- Summary

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Enhancing quantum state discrimination with indefinite causal order

Spiros Kechrimparis*, James Moran, Athena Karsa, Changhyoup Lee and Hyukjoon Kwon

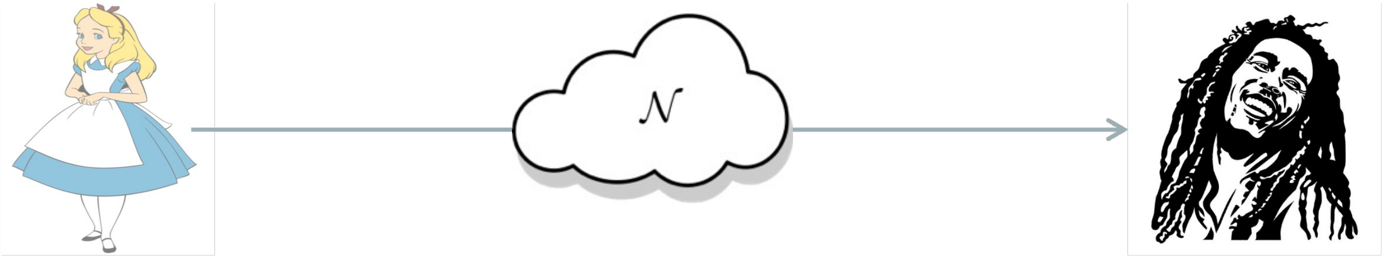
New Journal of Physics, Volume 26, December 2024

Citation Spiros Kechrimparis *et al* 2024 *New J. Phys.* **26** 123030

MOTIVATION

- Indefinite causal order (ICO) has shown advantages on several tasks.
- Particularly, many applications in quantum communication.
- State discrimination can be seen as a communication scenario.

QUANTUM COMMUNICATION



$\Omega = \{q_i, \rho_i\}_{i=1}^{n}$ $M = \{M_i\}_{i=1}^{n}$

- A communication scenario between two parties, a sender (A) and a receiver (B), consists of the following stages:
  1. A encodes characters from an alphabet into an ensemble of quantum states $\Omega = \{q_i, \rho_i\}_{i=1,\ldots,n}$.

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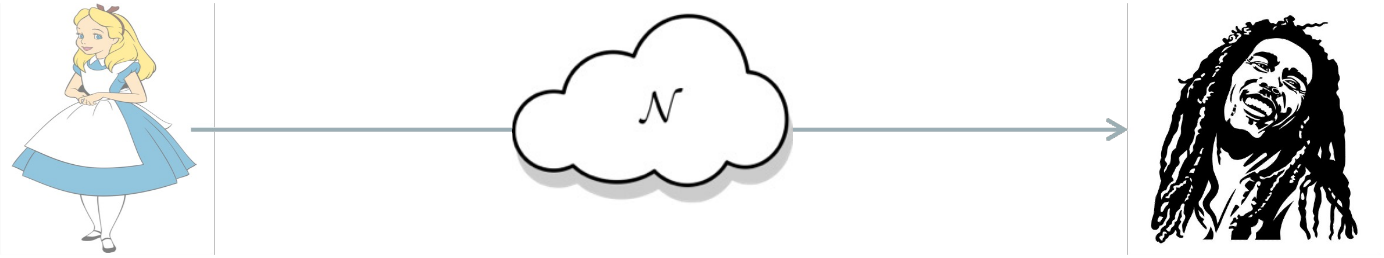

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  2. A transmits the states to B through a channel $\mathcal{N}$.

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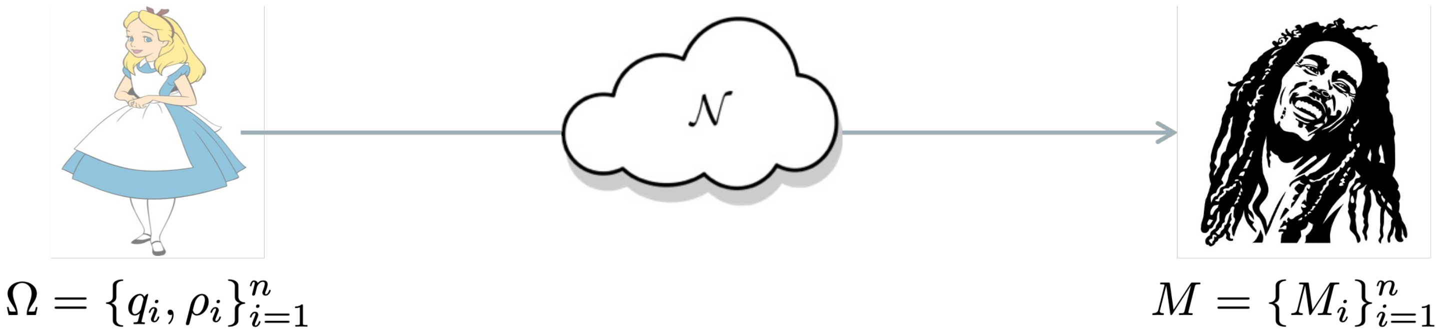

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  1. A encodes characters from an alphabet into an ensemble of quantum states $\Omega = \{q_i, \rho_i\}_{i=1,\ldots,n}$.
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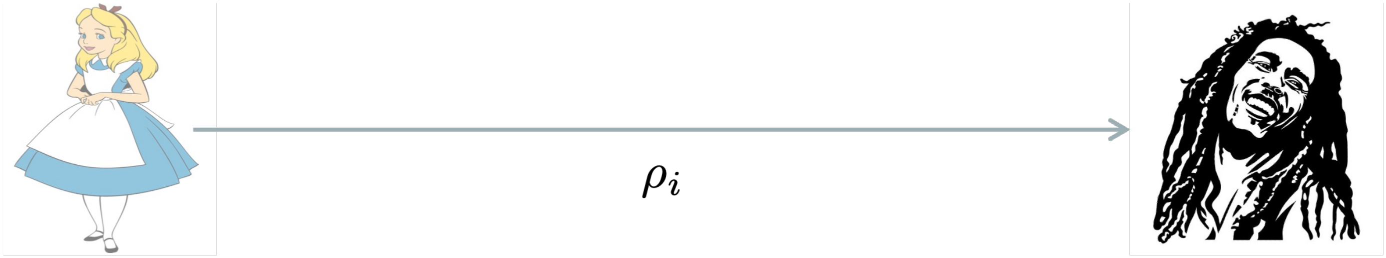
  3. B decodes the information by performing measurements on the received state to guess its label.

MINIMUM-ERROR STATE DISCRIMINATION



$\Omega = \{q_i, \rho_i\}_{i=1}^{n}$ $\quad$ $M = \{M_i\}_{i=1}^{n}$

- Standard MED is a scenario of quantum communication but with some differences:
  1. A encodes characters from an alphabet into a pre-agreed and **fixed** ensemble of states $\Omega = \{q_i,\ \rho_i\}_{i=1,\ldots,n}$.
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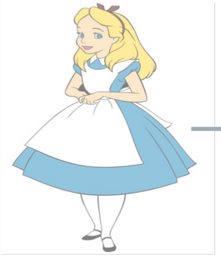

ρ_i

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  2. A transmits the state to B through a **noiseless** channel.
  3. B decodes the information by performing measurements on the received state to guess its label.
- The success of the task is given by the *guessing probability*:

$$p_g = \max_M \sum_i q_i \mathrm{tr}\left(M_i \rho_i\right)$$

subject to $M = \{M_i\}_{i=1,\ldots,n}$ being a POVM.

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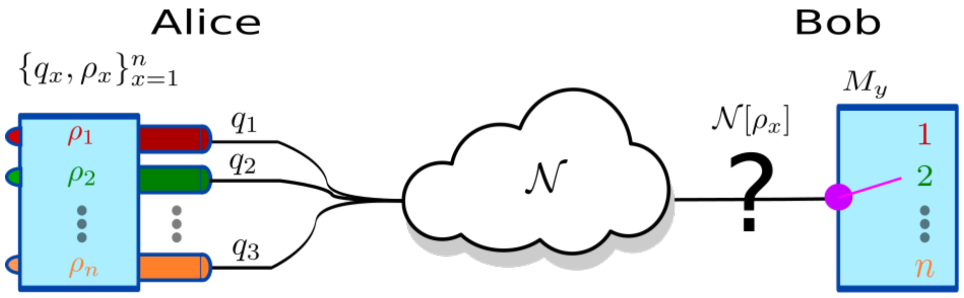
- For qubit states, it is known that a measurement with at most four non-null elements can achieve the optimal guessing.
- Recently an algorithmic process to find optimal measurements for qubit states has been derived.
- Necessary and sufficient conditions exist:

$$\sum_i q_i\rho_i M_i - q_j\rho_j \geq 0\,, \quad \forall j.$$

NOISY MINIMUM-ERROR STATE DISCRIMINATION



- In practice, often noise exists between A and B. The channel $\mathcal{N}$ effectively changes the ensemble as

$$\mathcal{N} \ : \ \Omega = \{q_i, \rho_i\}_{i=1}^n \ \to \ \Omega^{(\mathcal{N})} = \{q_i, \mathcal{N}(\rho_i)\}_{i=1}^n .$$

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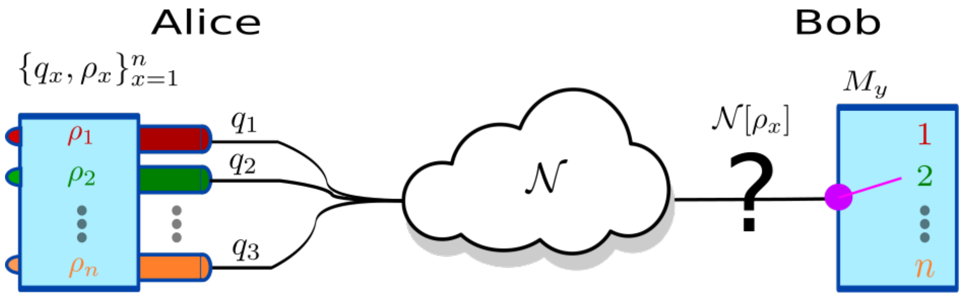

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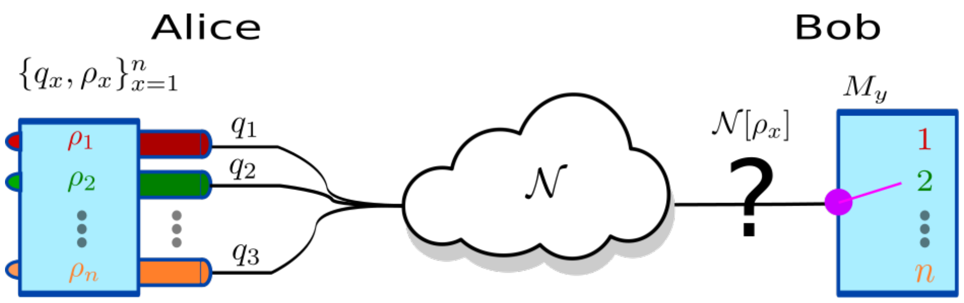

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- Since $\Omega^{(\mathcal{N})}$ is different than Ω, the original optimal measurement is no longer optimal in general.
- Two options:
  i. Quantum state tomography to find $\mathcal{N}(\rho_i)$
  ii. Channel tomography to identify $\mathcal{N}$.

OPTIMAL MEASUREMENT PRESERVING CHANNELS

- When do two channels $\mathcal{E}$ and $\mathcal{F}$ share an optimal measurement M?

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Preserving measurements for optimal state discrimination over quantum channels

Spiros Kechrimparis[1], Tanmay Singal[1], Chahan M. Kropf[2,3], and Joonwoo Bae[4,*]

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Published 21 December 2020 •

New Journal of Physics, Volume 22, December 2020

Citation Spiros Kechrimparis and Joonwoo Bae 2020 *New J. Phys.* **22** 123024

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- The depolarization channel

$$D_\mu(\rho) = (1-\mu)\rho + \mu\, \mathbb{I}/d\,,\ \mu \in [0,1]\,,$$

is OMP for ensembles:

i. $\Omega_0 = \{1/n,\ \rho_i\}_{i=1,\dots,n}$ of n states with equal *a priori* probabilities.

ii. $\Omega_2 = \{q_i,\ \rho_i\}_{i=1,2}$ of two states appearing with arbitrary *a priori* probabilities.

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Lucien Hardy

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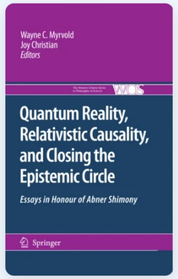

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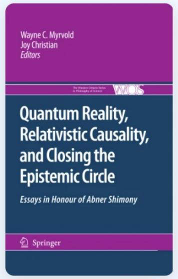

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Chiranjib Mukhopadhyay and Arun Kumar Pati

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Journal of Physics Communications, Volume 4, Number 10

Citation Chiranjib Mukhopadhyay and Arun Kumar Pati 2020 *J. Phys. Commun.* **4** 105003

DOI 10.1088/2399-6528/abbd77

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Quantum Metrology with Indefinite Causal Order

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Giulio Chiribella, Manik Banik, Some Sankar Bhattacharya, Tamal Guha, Mir Alimuddin, Arup Roy, Sutapa Saha, Sristy Agrawal and Guruprasad Kar

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Lee A. Rozema ✉, Teodor Strömberg, Huan Cao, Yu Guo, Bi-Heng Liu & Philip Walther ✉

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THE QUANTUM SWITCH

Quantum computations without definite causal structure

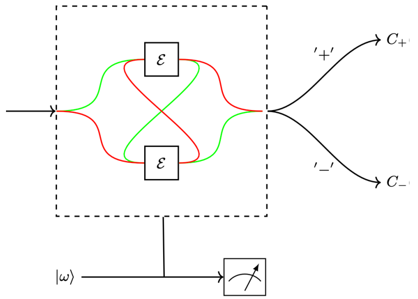
Giulio Chiribella[1,*], Giacomo Mauro D'Ariano[2,†], Paolo Perinotti[2,‡], and Benoit Valiron[3,§]

Phys. Rev. A **88**, 022318 – **Published 14 August, 2013**

DOI: https://doi.org/10.1103/PhysRevA.88.022318

- The *quantum switch* is a supermap that superposes the ordering of applying two channels $\mathcal{E}$ and $\mathcal{F}$. Explicitly:

$$S_\omega(\mathcal{E},\mathcal{F}) = \sum_{i,j} K_{ij}(\rho \otimes \omega) K_{ij}^\dagger \ ,$$



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$$K_{ij} = E_i F_j \otimes |0\rangle\langle 0|_C + F_j E_i \otimes |1\rangle\langle 1|_C \ .$$

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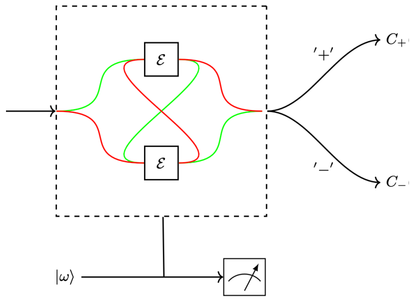

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$$\mathcal{S}_{|+\rangle\langle+|}(\mathcal{E},\mathcal{E})(\rho) = \frac{1}{4}\sum_{i,j} \{E_i, E_j\}\,\rho\,\{E_i, E_j\}^\dagger \otimes |+\rangle\langle+| + \frac{1}{4}\sum_{i,j} [E_i, E_j]\,\rho\,[E_i, E_j]^\dagger \otimes |-\rangle\langle-| \ ,$$

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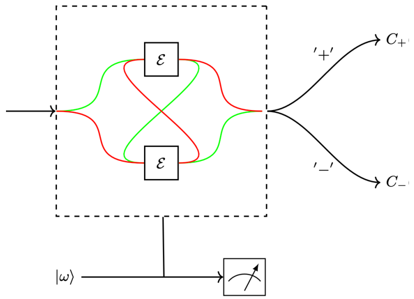

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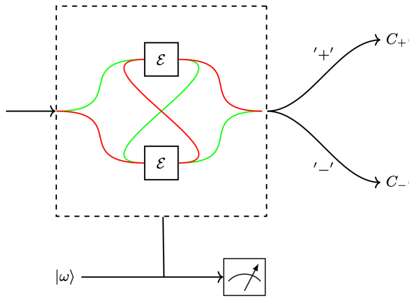

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$$S_{|+\rangle\langle+|}(\mathcal{E},\mathcal{E}) = q_+C_+(\rho)\otimes|+\rangle\langle+| + q_-C_-(\rho)\otimes|-\rangle\langle-| \;,$$

where the channels C_+, C_- are

$$C_+(\rho) = \frac{(p_0^2+p_1^2+p_2^2+p_3^2)\rho + 2p_0(p_1X\rho X + p_2Y\rho Y + p_3Z\rho Z)}{q_+}\,,$$

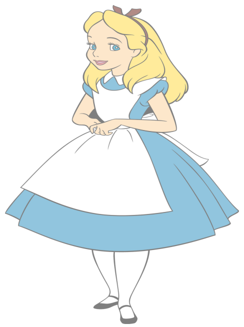
$$C_-(\rho) = \frac{2p_1p_2Z\rho Z + 2p_2p_3X\rho X + 2p_3p_1Y\rho Y}{q_-}\,,$$

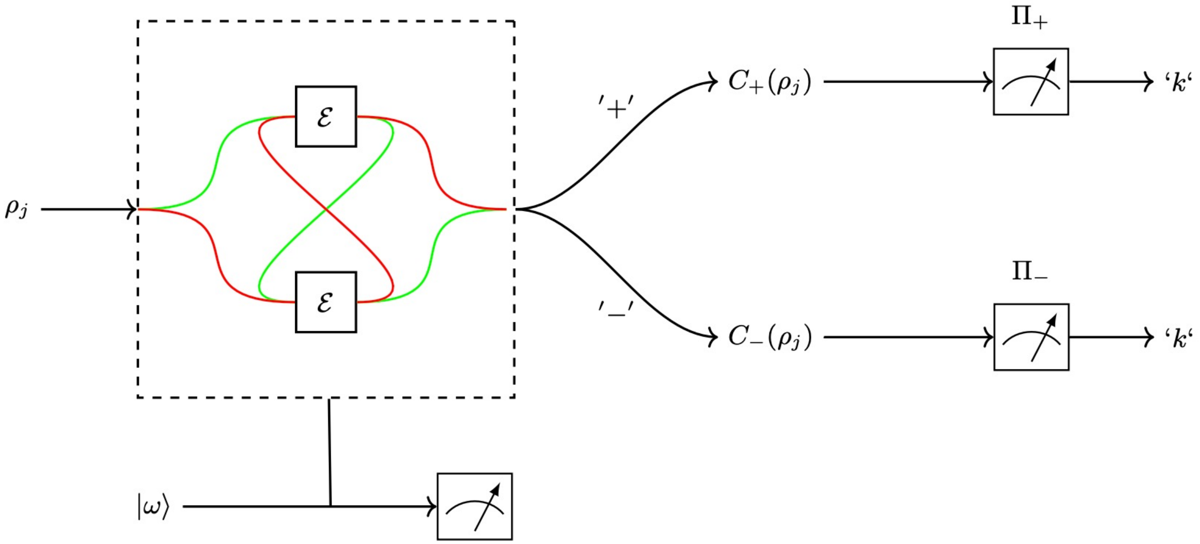
and the probabilities are

$$q_- = 2(p_1p_2 + p_2p_3 + p_3p_1)\,,\quad q_+ = 1 - q_- \,.$$

QUANTUM SWITCH AND STATE DISCRIMINATION

- A pictorial representation of the protocol is:





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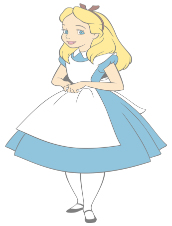
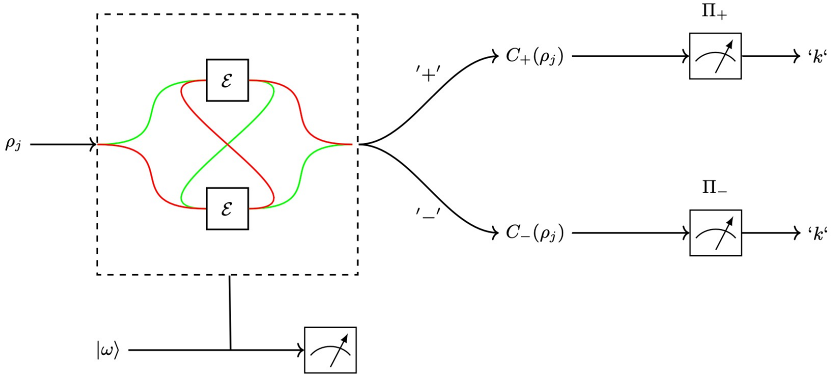
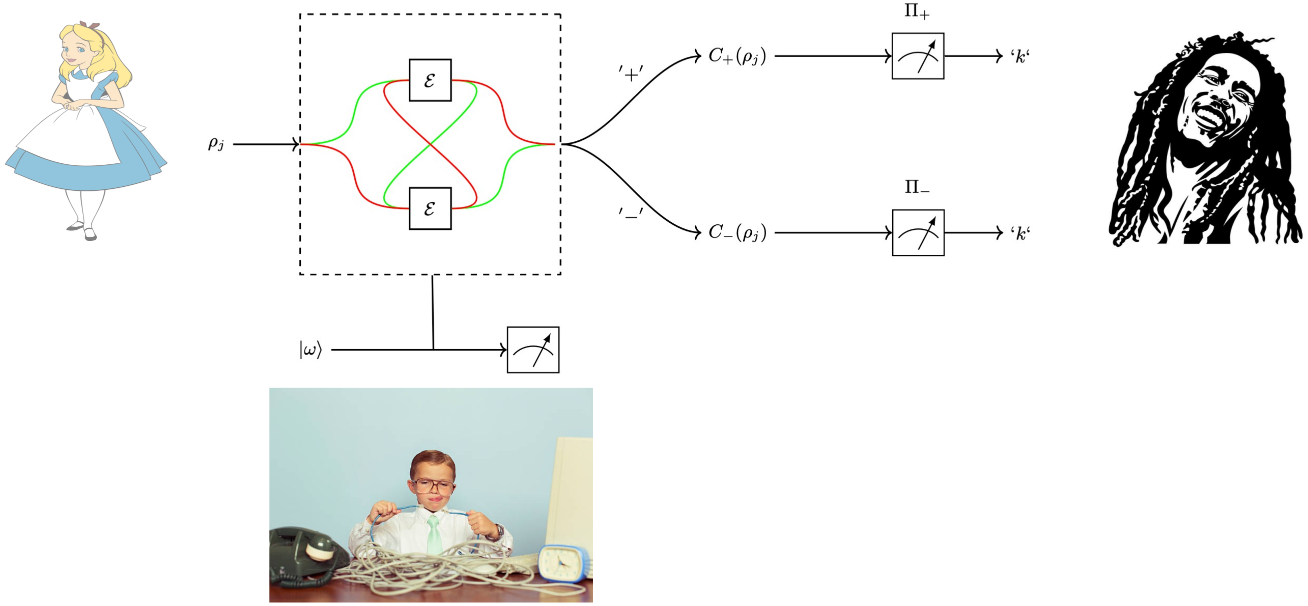
    1. The sender prepares a state ρ_j and sends it to the communication provider.





QUANTUM SWITCH AND STATE DISCRIMINATION

- In detail the scenario works as follows:
    1. The sender prepares a state ρ_j and sends it to the communication provider.
    2. The communication provider implements the quantum switch, measures the ancilla qubit and communicates the outcome to the receiver.



QUANTUM SWITCH AND STATE DISCRIMINATION

- In detail the scenario works as follows:
  1. The sender prepares a state ρ_j and sends it to the communication provider.
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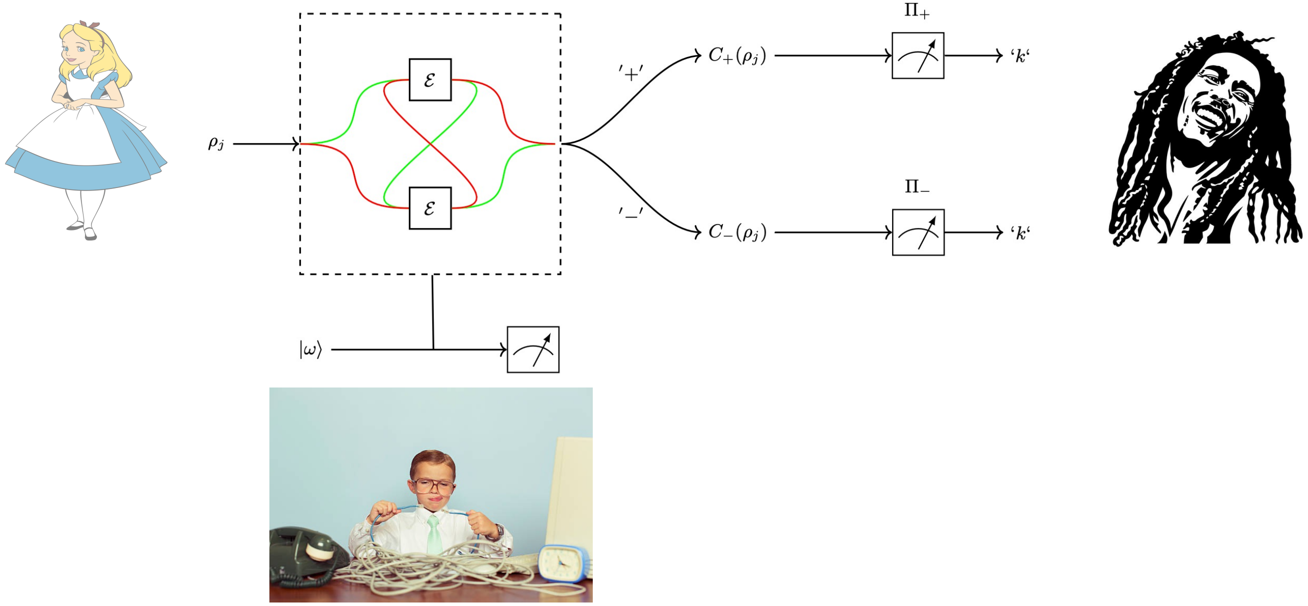
  3. The receiver applies an appropriate measurement Π_+, Π_- and guesses the label of the received state.



QUANTUM SWITCH AND STATE DISCRIMINATION

- There are two scenarios in which the quantum switch can assist:
  1. Ω and $\mathcal{E}$ such that C_+, C_- are OMP or the new optimal measurement can be easily inferred. In such case, the advantage can be twofold:
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- In both cases, applying the optimal measurements for C_+ and C_- , we obtain the average guessing:

$$p_g^{(\mathcal{S})} = q_+ p_g^+ + q_- p_g^- \ .$$

APPLICATIONS OF THE PROTOCOL

- An example is the depolarisation channel:

$$\mathcal{D}_p(\rho) = \left(1 - \frac{3p}{4}\right)\rho + \frac{p}{4}\left(X\rho X + Y\rho Y + Z\rho Z\right)\,,\; p \in [0, {}^{4}\!/\!{}_{3}]\,.$$

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- If no information on the value of p, we do not know what measurement to apply.
- The action of the quantum switch in this case gives:

$$C_+(\rho) = \mathcal{D}_{\tilde{p}}(\rho)\,,\quad \tilde{p} = \frac{4(4-3p)p}{8-3p^2}\,,$$

$$C_-(\rho) = \mathcal{D}_{4/3}\,.$$

with

$$q_- = \frac{3p^2}{8}\,, q_+ = 1 - \frac{3p^2}{8}\ .$$

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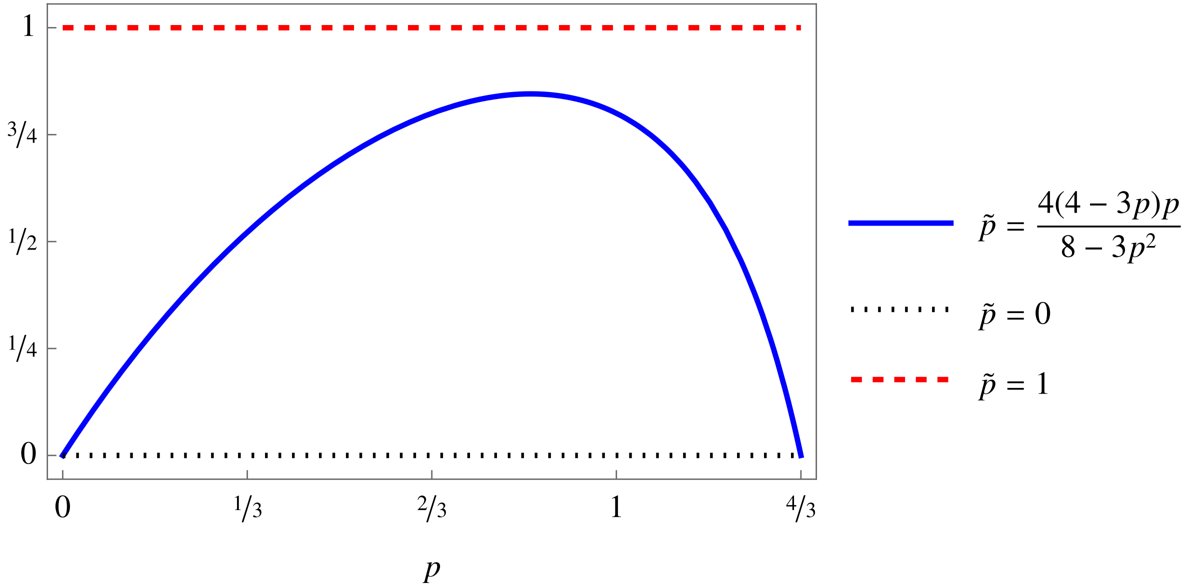

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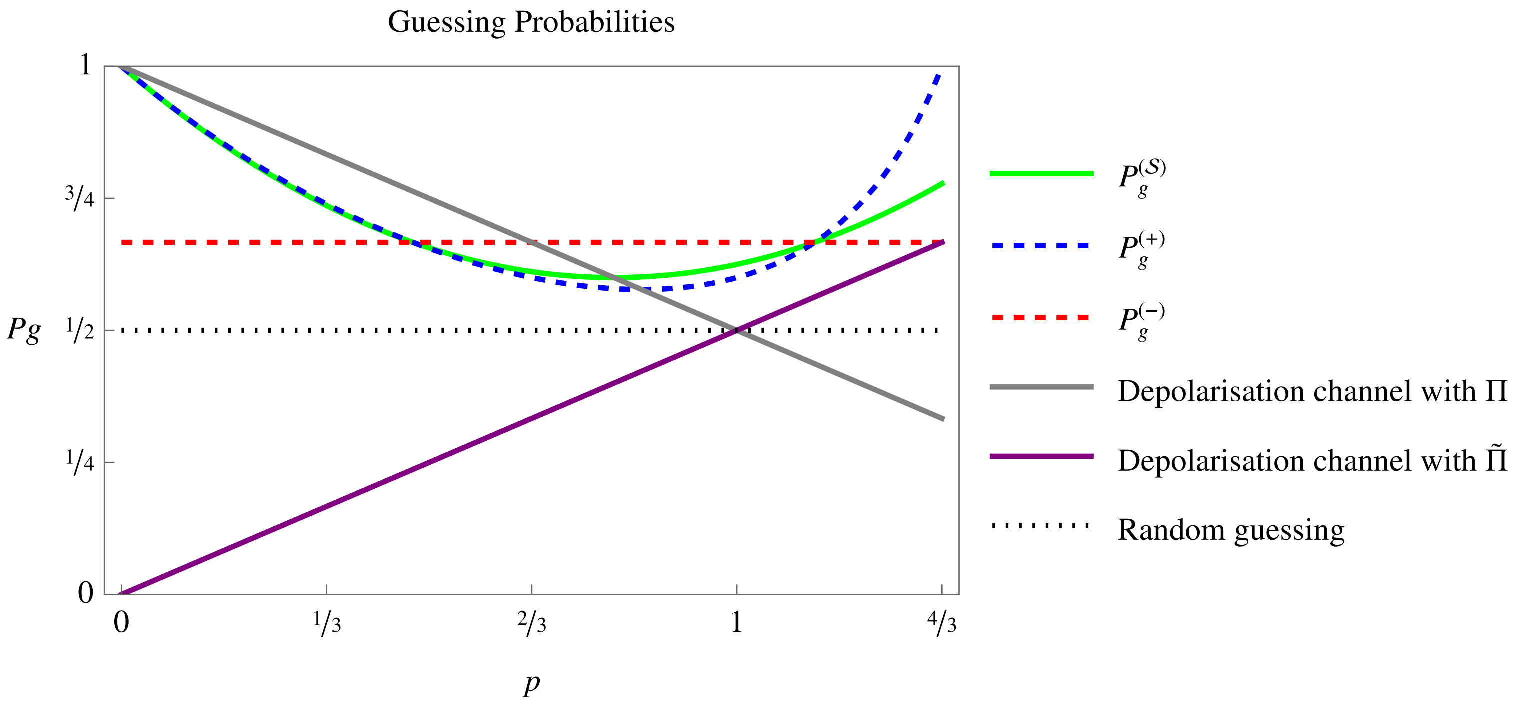

APPLICATIONS OF THE PROTOCOL

Result 1 *For any value of $p > {}^4\!/_5$, the discrimination protocol with the quantum switch leads to a higher guessing probability than can be achieved using the channel. Interestingly, at $p = 1$ the depolarisation channel sends all states to the maximally mixed one, removing any possibility of guessing better than uniform, i.e. $p_g = {}^1\!/_n$, while the quantum switch allows for a correct detection with a probability of*

$$p_g^{(\mathcal{S})} = \frac{3 + np_g}{4n} \,. \tag{1}$$

APPLICATIONS OF THE PROTOCOL

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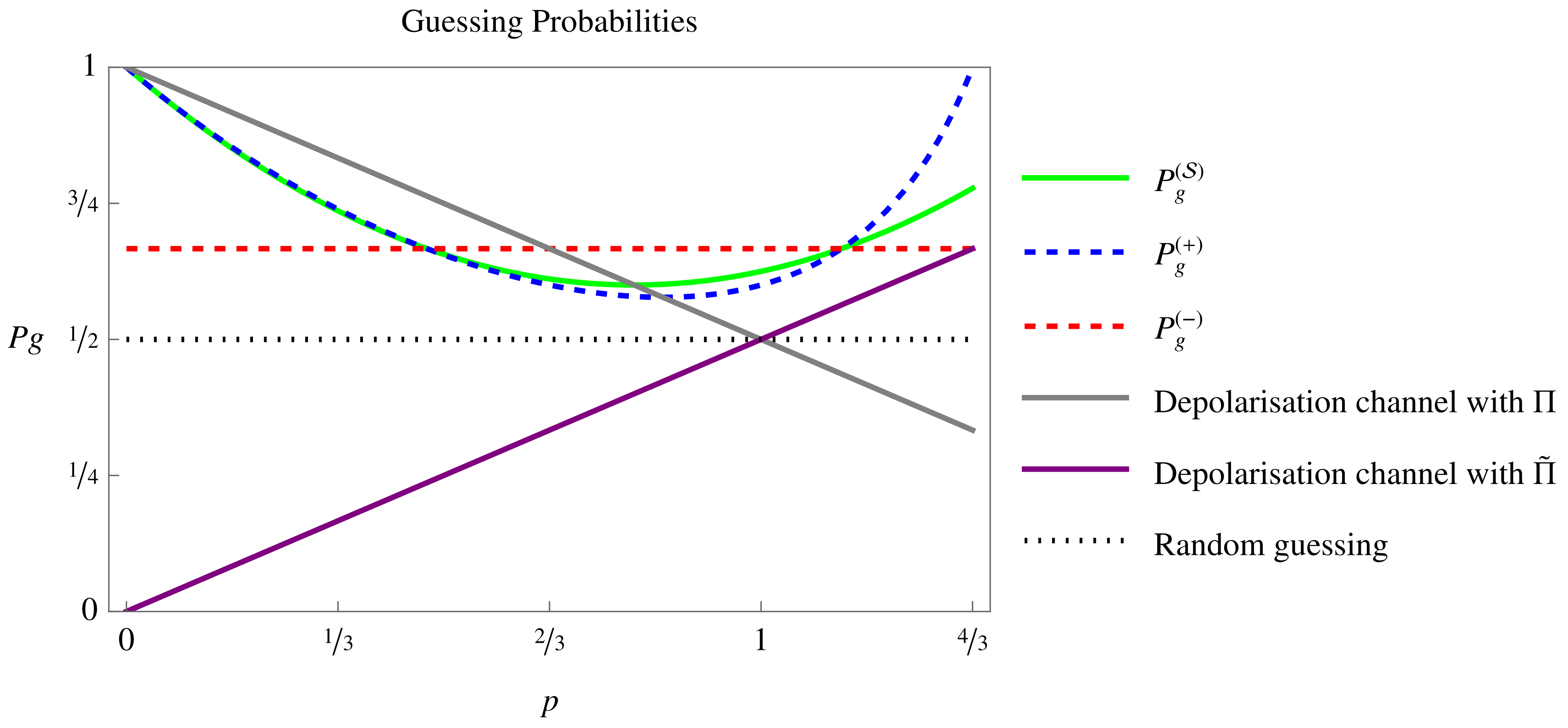

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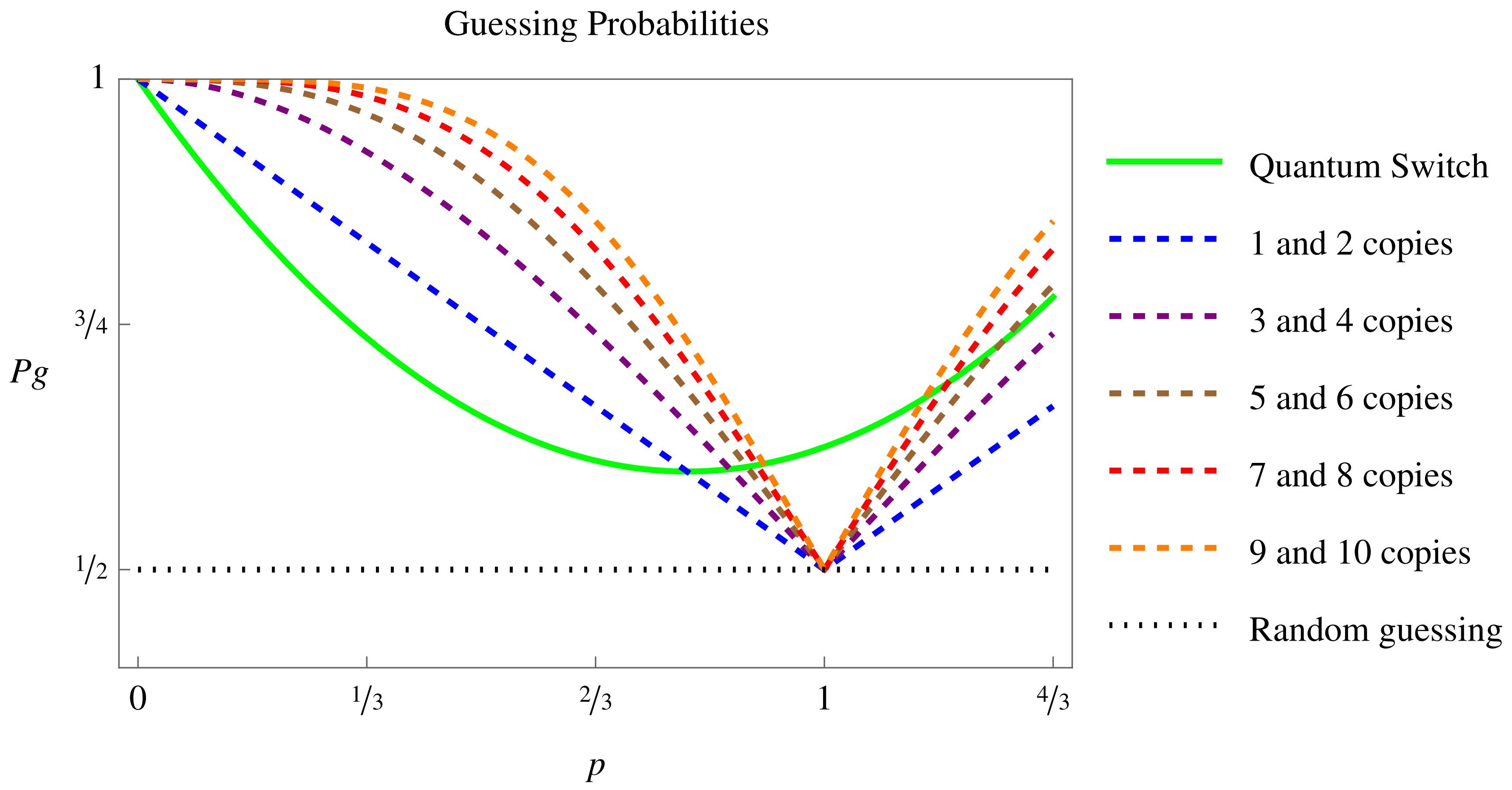


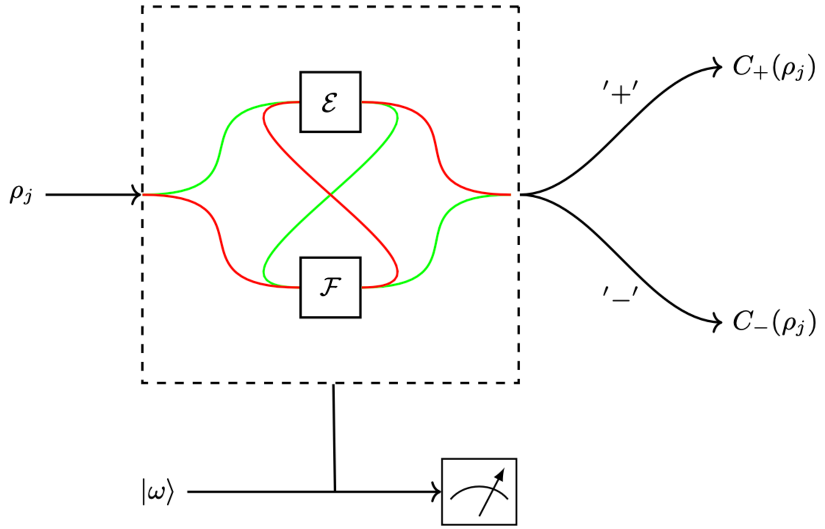
APPLICATIONS OF THE PROTOCOL

Result 2 *For any finite number m of copies of the state and the channel, there is a region in the parameter space of the depolarisation channel around the value $p = 1$ for which quantum state discrimination with the quantum switch achieves higher guessing probability than the multiple-copy discrimination scenario.*

HIGHER-ORDER SWITCHES

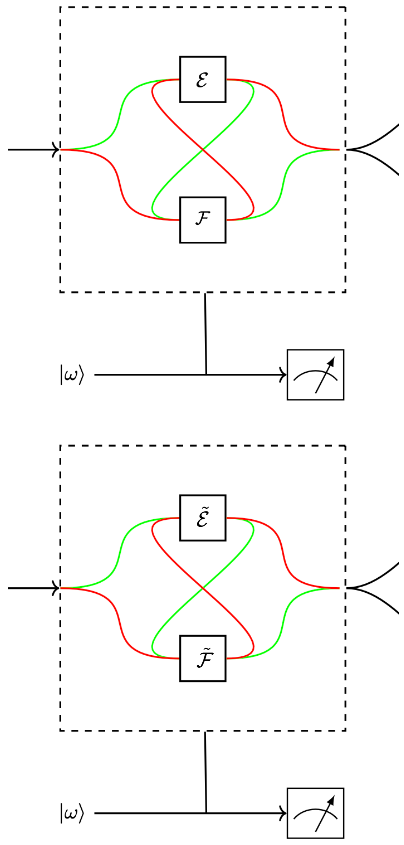
- The quantum switch acts as

$$\mathcal{S}_\omega(\mathcal{E},\mathcal{F})(\rho) = \sum_{i,j}(K_{ij}\rho\otimes\omega)K_{ij}^\dagger = \frac{1}{4}\sum_{i,j}\{E_i,F_j\}\,\rho\,\{E_i,F_j\}^\dagger\otimes\omega + \frac{1}{4}\sum_{i,j}[E_i,F_j]\,\rho\,[E_i,F_j]^\dagger\otimes Z\omega Z\,.$$



HIGHER-ORDER SWITCHES

- However, one can envisage a switch of switches, or *superswitch,*



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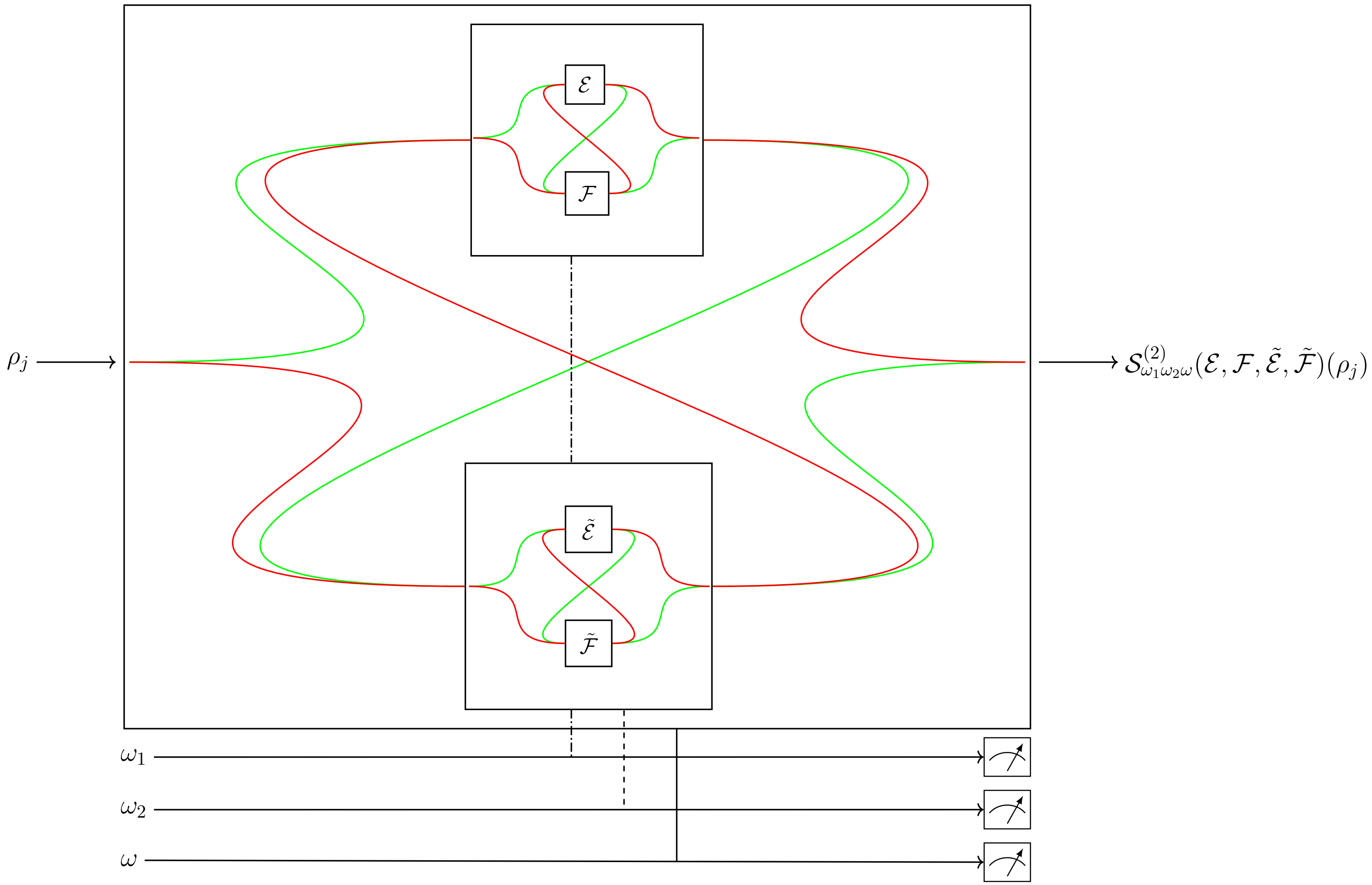

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- These effectively give nested expressions of anticommutators and commutators.
- The expressions of the $(n+1)$-order superswitch can be efficiently derived from the expressions of the n-order superswitch through recurrence relations that can be iterated.

HIGHER-ORDER SWITCHES

If $\mathcal{E} = \mathcal{F} = \ldots = \mathcal{P}_{\vec{v}}$ and denote by $C_s^{(n)}$ and $r_s^{(n)}$ the channels and respective probabilities of the n-th order superswitch, the channels of the $(n+1)$-order superswitch are

$$C_{ss'+}^{(n+1)} = \frac{\mathfrak{a}(C_s^{(n)}, C_{s'}^{(n)})}{r_{ss'+}^{(n+1)}}\,, \; C_{ss'-}^{(n+1)} = \frac{\mathfrak{c}(C_s^{(n)}, C_{s'}^{(n)})}{r_{ss'-}^{(n+1)}}\,,$$
$$r_{ss'+}^{(n+1)} = \Pr\left(\mathfrak{a}(C_s^{(n)}, C_{s'}^{(n)})\right)\,, \; r_{ss'-}^{(n+1)} = \Pr\left(\mathfrak{c}(C_s^{(n)}, C_{s'}^{(n)})\right)\,, \tag{1}$$

Result 3 *Any n-order superswitch can be analytically evaluated by iterating Eqs.* (1) *under the initial conditions* $C^{(0)} = \mathcal{E}$ *and* $r^{(0)} = 1$.

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where $\vec{v}_i = \{\alpha_i, \beta_i, \gamma_i, \delta_i\}\,, i = 1, 2$, and

$$\mathfrak{a}(\mathcal{E}, \mathcal{F}) \equiv \mathfrak{a}(\vec{v}_1, \vec{v}_2) = \{\alpha_1\alpha_2 + \beta_1\beta_2 + \gamma_1\gamma_2 + \delta_1\delta_2\,, \alpha_1\beta_2 + \beta_1\alpha_2\,, \alpha_1\gamma_2 + \gamma_1\alpha_2\,, \alpha_1\delta_2 + \delta_1\alpha_2\}\,,$$
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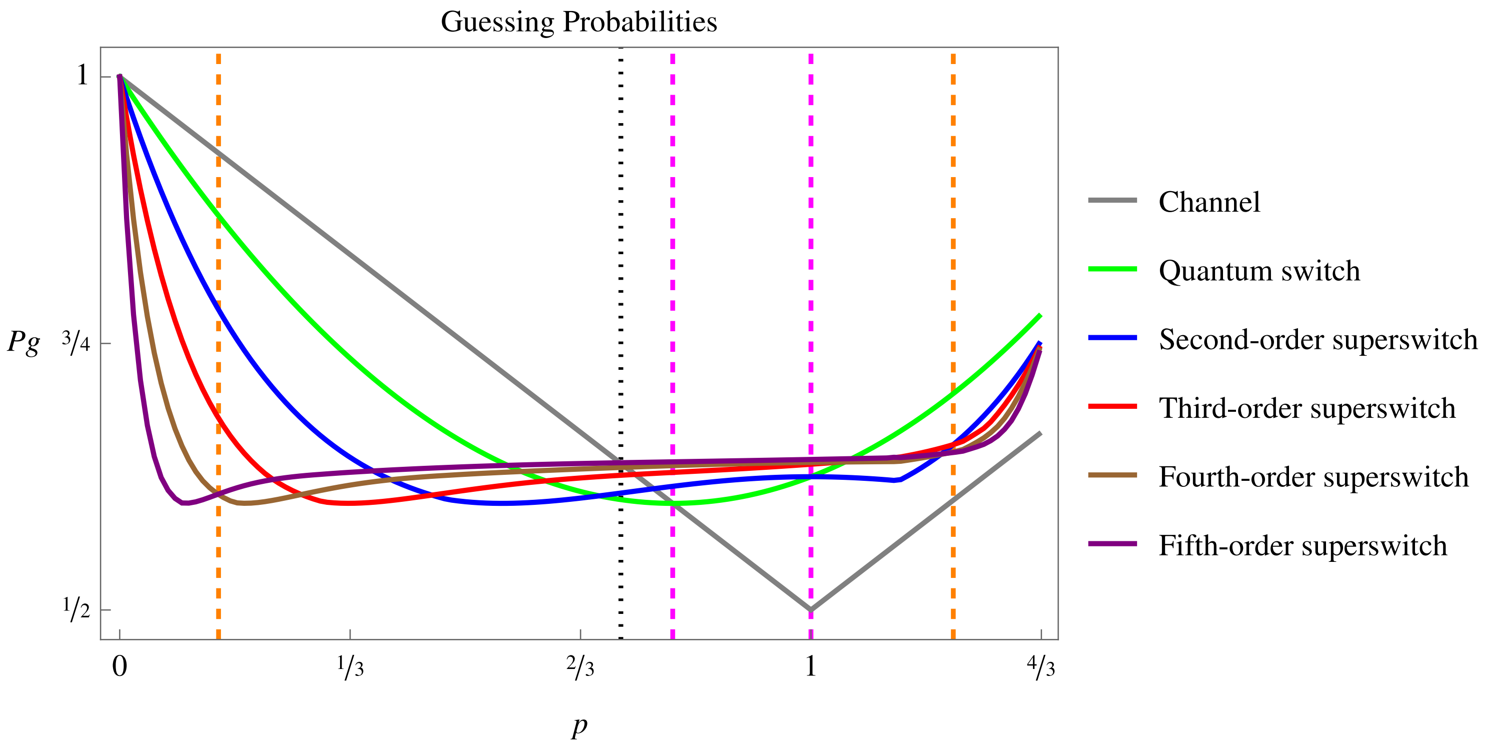
HIGHER-ORDER SWITCHES



Result 4 *There is a region in the parameter space of the depolarisation channel for which the higher the order of the superswitch, the higher the guessing probability. Moreover, as a consequence of Results 1 and 2, the guessing probability in a region including the point $p = 1$ increases with the order of the superswitch in comparison to the multiple-copy guessing probability for any finite number of copies.*

HIGHER-ORDER SWITCHES

- Let $\Omega_2 = \{q_i, |i\rangle\langle i| \}_{i=0,1}$ be an ensemble of two orthogonal states and $\mathcal{E}_{\vec{p}} = p_0\rho + p_1 X\rho X + p_2 Y\rho\, Y + p_3 Z\rho Z$ an arbitrary Pauli channel.

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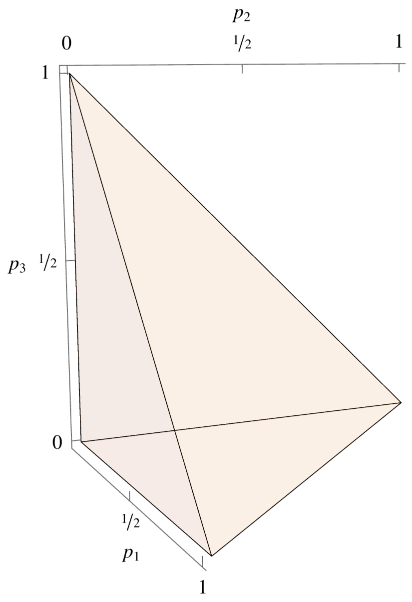

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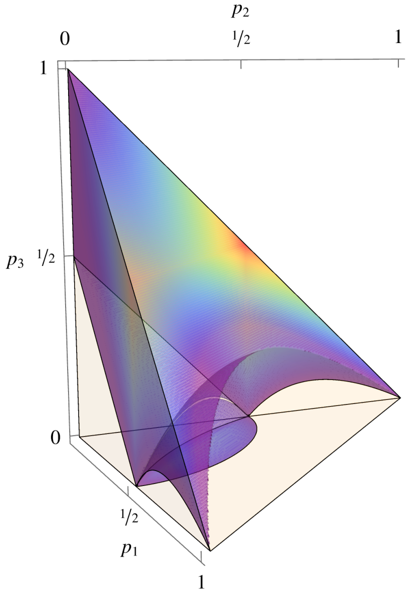

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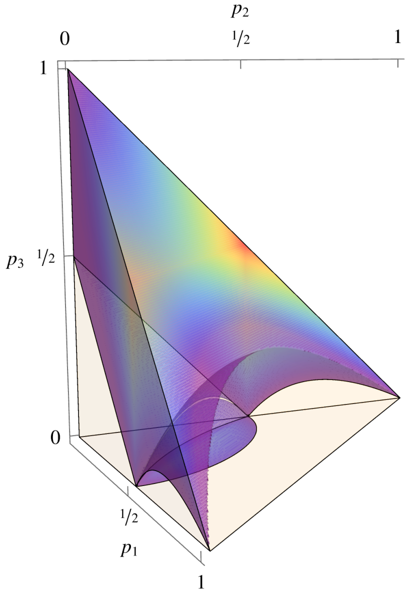

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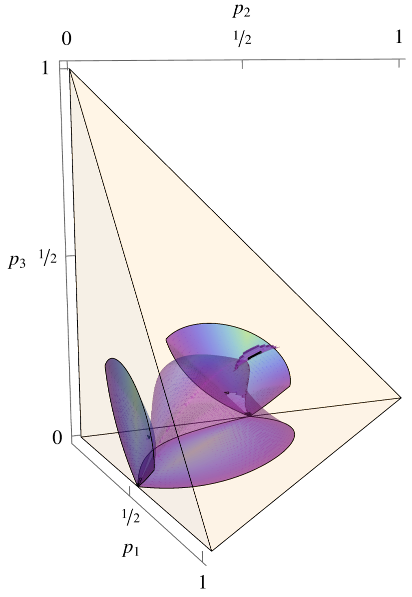

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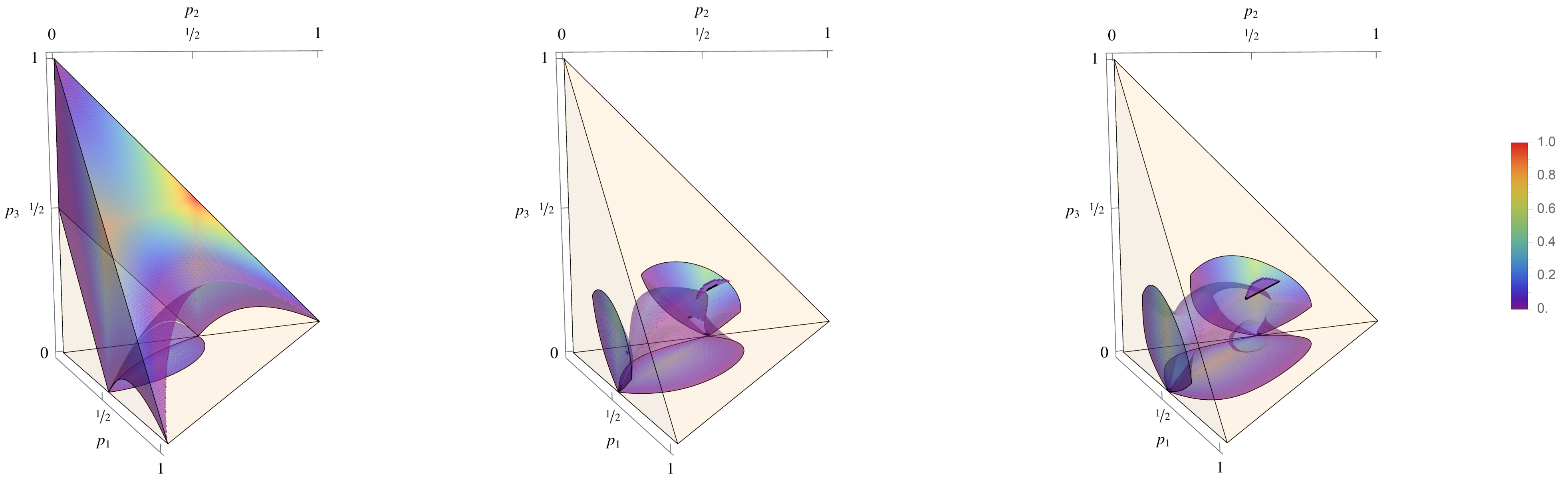

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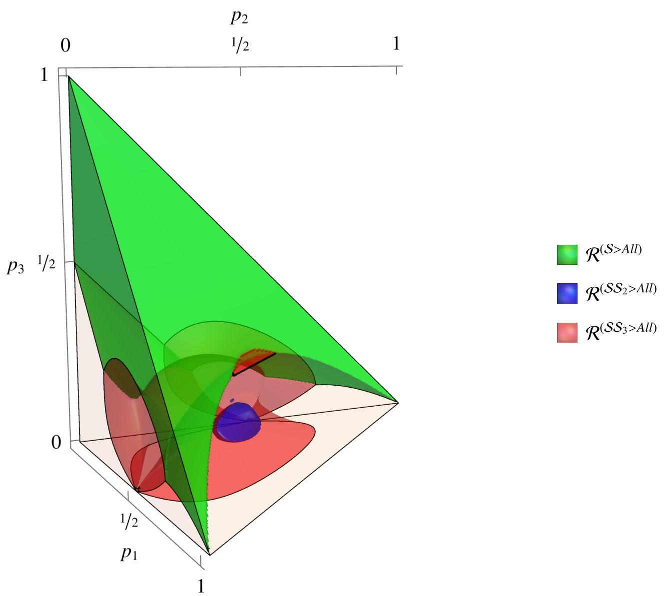

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