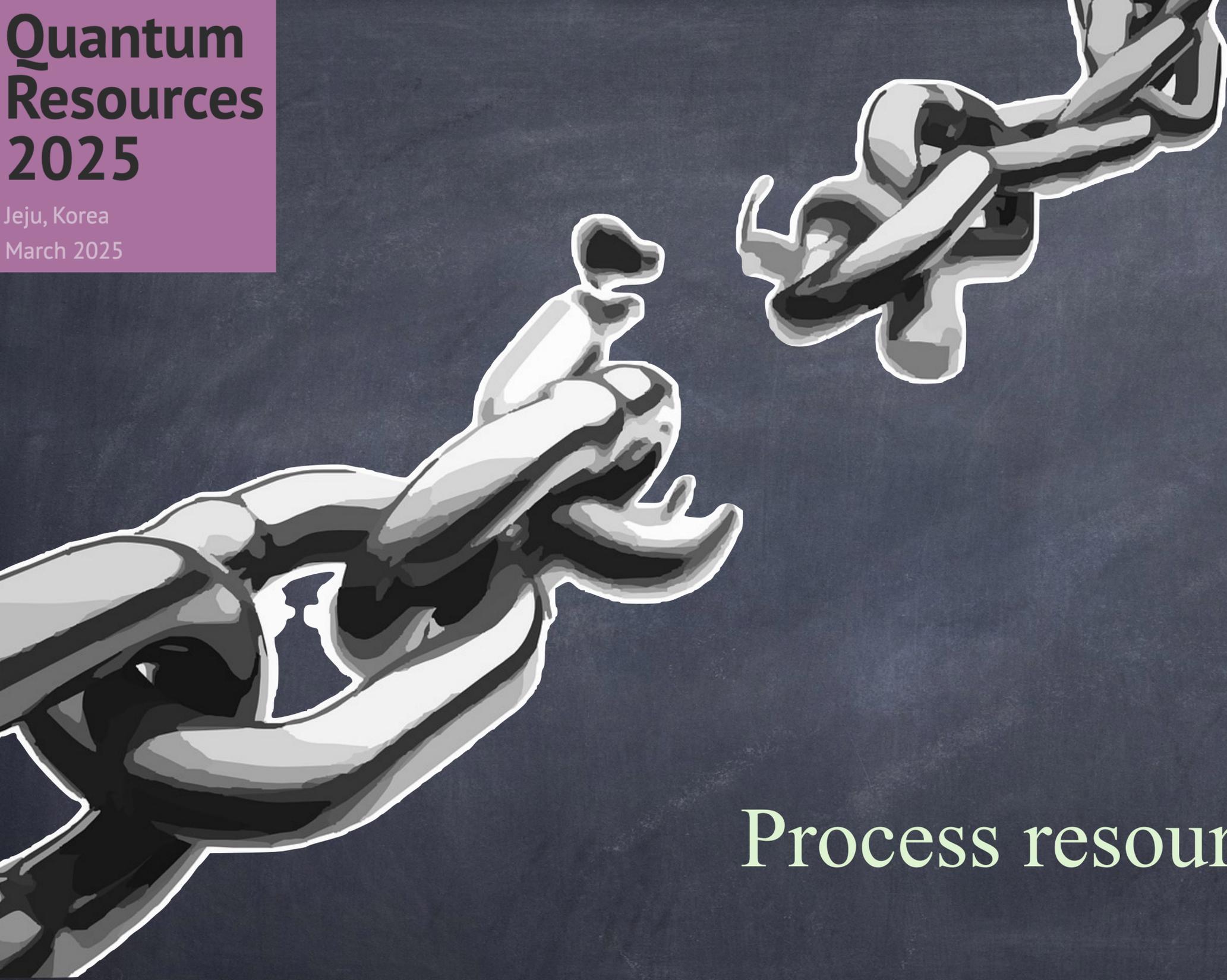


Quantum Resources 2025

Jeju, Korea
March 2025



Process resource-breaking channels

Rivu Gupta 20/03/2025

with Ayan Patra, Alessandro Ferraro, and Aditi Sen (De)

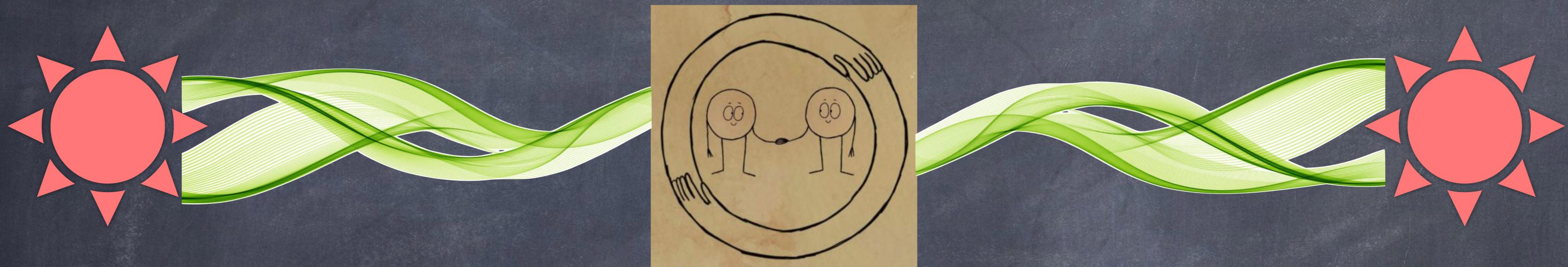
**Quantum
Resources
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Process resource-breaking channels the case for magic in universal quantum computation

Rivu Gupta 20/03/2025
with Ayan Patra, Alessandro Ferraro, and Aditi Sen (De)

Resource breaking channels

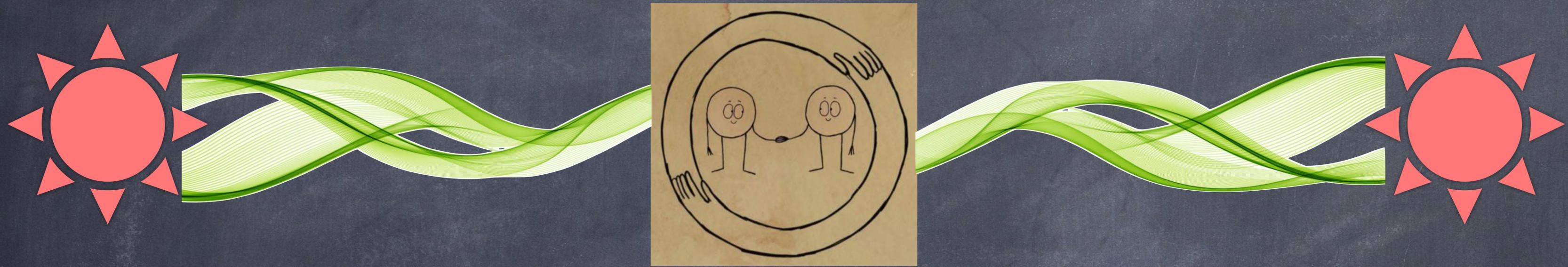


Shared quantum property :

Entanglement \longrightarrow **communication, key distribution, MBQC**

[Rev. Mod. Phys. 81, 865 \(2009\)](#)

Resource breaking channels



Shared quantum property :

Entanglement \longrightarrow **communication, key distribution, MBQC** [Rev. Mod. Phys. 81, 865 \(2009\)](#)

Non-locality \longrightarrow **communication complexity, device independence, randomness generation**
[Rev. Mod. Phys. 86, 419 \(2014\)](#)

Resource breaking channels



“Broken” quantum property :

~~Entanglement~~ → ~~communication, key distribution, MBQC~~ [Rev. Math. Phys. 15, 629 \(2003\)](#)

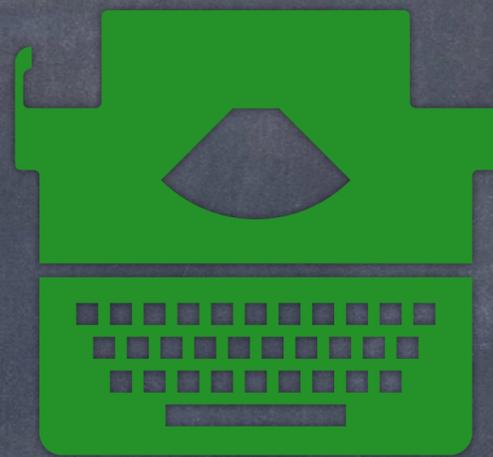
~~Non-Local~~ → ~~communication complexity, device independence, randomness~~
~~generation~~
[J. Phys. A: Math. Theor. 48 155302 \(2015\)](#)

Process resource-breaking channels



ρ

Resource state



Information theoretic
task

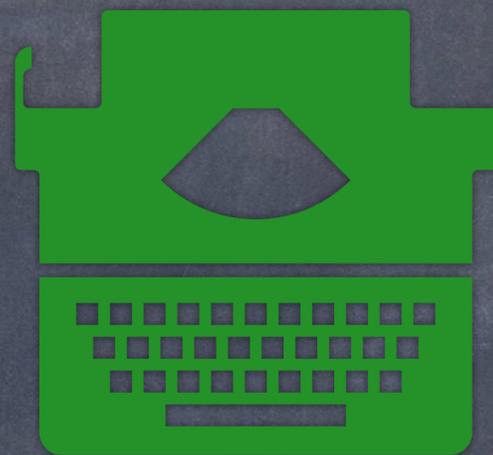


Quantum
advantage

Process resource-breaking channels



Resource state



Information theoretic
task



Quantum
advantage

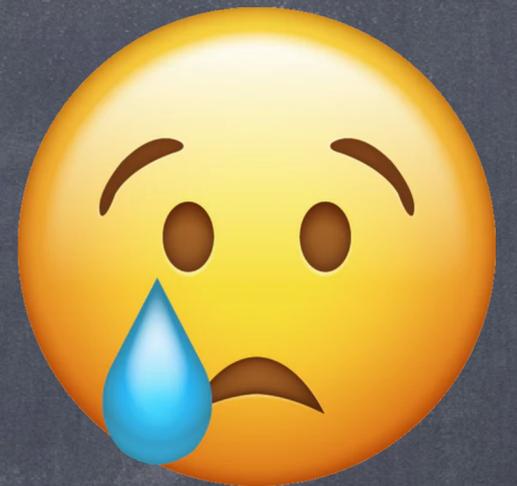
$$Q_{\rho}^{\mathcal{P}} > Q_{\text{cl}}^{\mathcal{P}} \text{ with } \mathcal{M}(\rho) > 0$$

e.g. $DCC > \log_2 d_A, F > \frac{2}{d+1}$

Phys. Rev. Lett. 69, 2881 (1992)

Phys. Rev. Lett. 70, 1895 (1993)

Process resource-breaking channels



$$(\Lambda_{\text{PBT}} \otimes \mathbb{I})\rho$$

Resource state

Information theoretic
task

No quantum
advantage

$$Q_{\rho}^{\mathcal{P}} \leq Q_{\text{cl}}^{\mathcal{P}} \text{ with } \mathcal{M}(\rho) > 0$$

e.g. $DCC \leq \log_2 d_A, F \leq \frac{2}{d+1}$

[arxiv: 2309.03108](https://arxiv.org/abs/2309.03108)

A. Muhuri, A. Patra, R.G., A. Sen (De)

Process - Universal Quantum Computation

Process - Universal Quantum Computation



Non-stabilizerness

Process - Universal Quantum Computation



Non-stabilizerness

Magic



Process - Universal Quantum Computation

Hinder UQC \longrightarrow
Magic-breaking channels

[arxiv: 2409.04425](https://arxiv.org/abs/2409.04425)

A. Patra, R.G., A. Ferraro, A. Sen (De)



Bucket List

- Define magic-breaking channels (MB)
- Properties of MB independent of dimension
- Geometry of qubit channels (prerequisite)
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- Different classes of MB
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Magic-breaking channels : Definition

$$\mathcal{M} = \left\{ \Lambda^{\mathbb{C}^d \rightarrow \mathbb{C}^d} : \Lambda^{\mathbb{C}^d \rightarrow \mathbb{C}^d}(\rho) \in \text{STAB}(\mathbb{C}^d) \forall \rho \in \mathbb{C}^d \right\}$$

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convert any input state to stabilizer state at output



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convert any input state to stabilizer state at output



necessary for fault tolerance



efficiently simulable on
classical computers

Magic-breaking channels : Definition

$$\mathcal{M} = \left\{ \Lambda^{\mathbb{C}^d \rightarrow \mathbb{C}^d} : \Lambda^{\mathbb{C}^d \rightarrow \mathbb{C}^d}(\rho) \in \text{STAB}(\mathbb{C}^d) \forall \rho \in \mathbb{C}^d \right\}$$



Magic-breaking

$$\tilde{\mathcal{M}} = \left\{ \Lambda^{\mathbb{C}^d \rightarrow \mathbb{C}^d} : U_{NC} \circ \Lambda^{\mathbb{C}^d \rightarrow \mathbb{C}^d}(\rho) \in \text{STAB}(\mathbb{C}^d) \forall \rho \in \mathbb{C}^d \right\}$$



Strictly magic-breaking

convert any input state to stabilizer state at output even with non-Clifford post-processing

(similar to absolute separable states [Phys. Rev. A. 63, 032307 \(2001\)](#))

Bucket List

- Define magic-breaking channels (MB)
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Properties

 Unitaries can never be MB

Properties

 Unitaries can never be MB

STAB is a discrete and finite set in any dimension

Unitaries involve infinitesimal rotations as well

=> cannot always transform any state to a stabilizer

Properties

Channels destroying magic of pure states can do so for all states

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i| \text{ and STAB is convex}$$

Properties



Channels destroying magic of pure states can do so for all states

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i| \text{ and STAB is convex}$$

Does not hold for breaking channels in non-convex resource theories

Properties



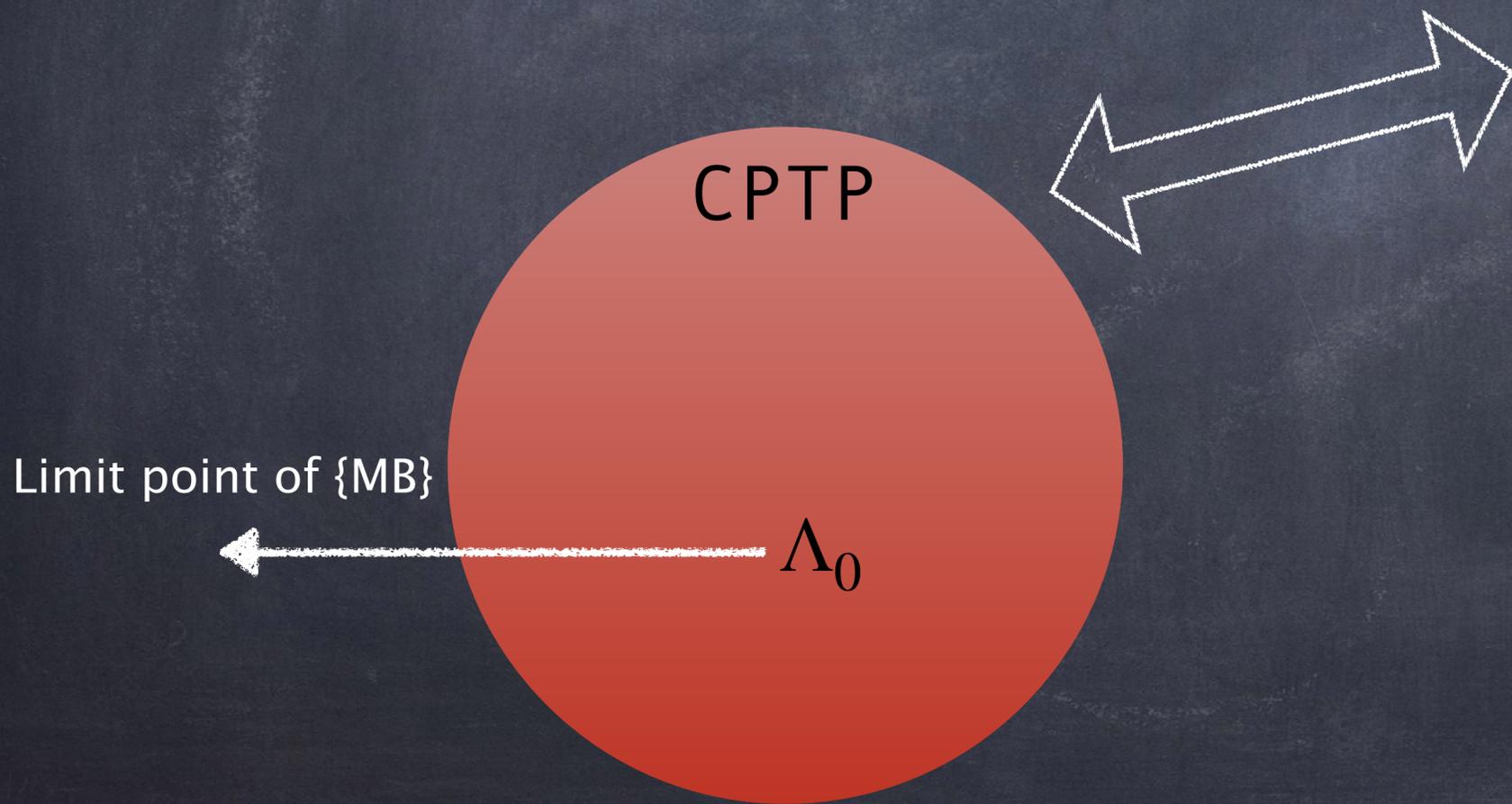
MB channels form a convex and compact set



STAB is convex

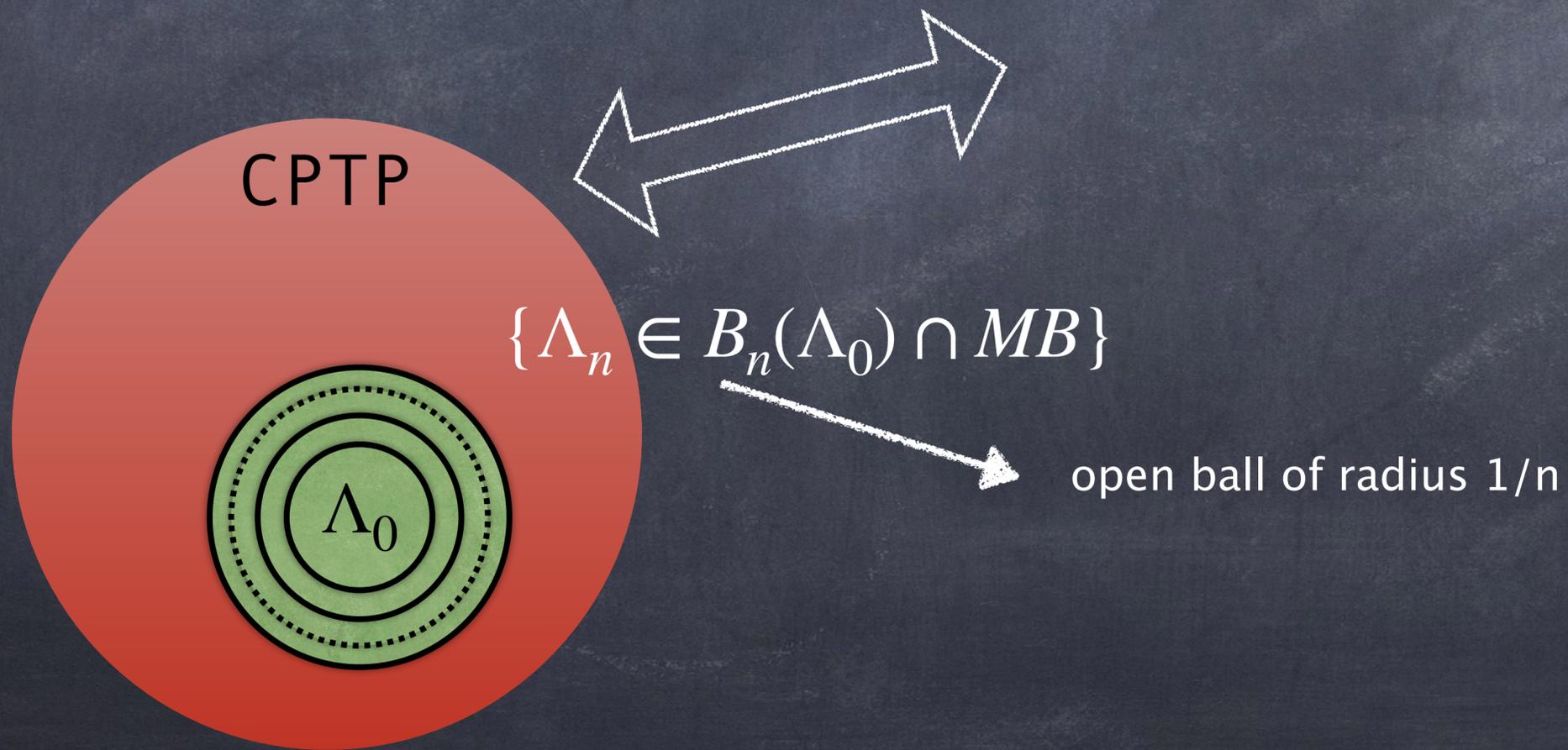
Properties

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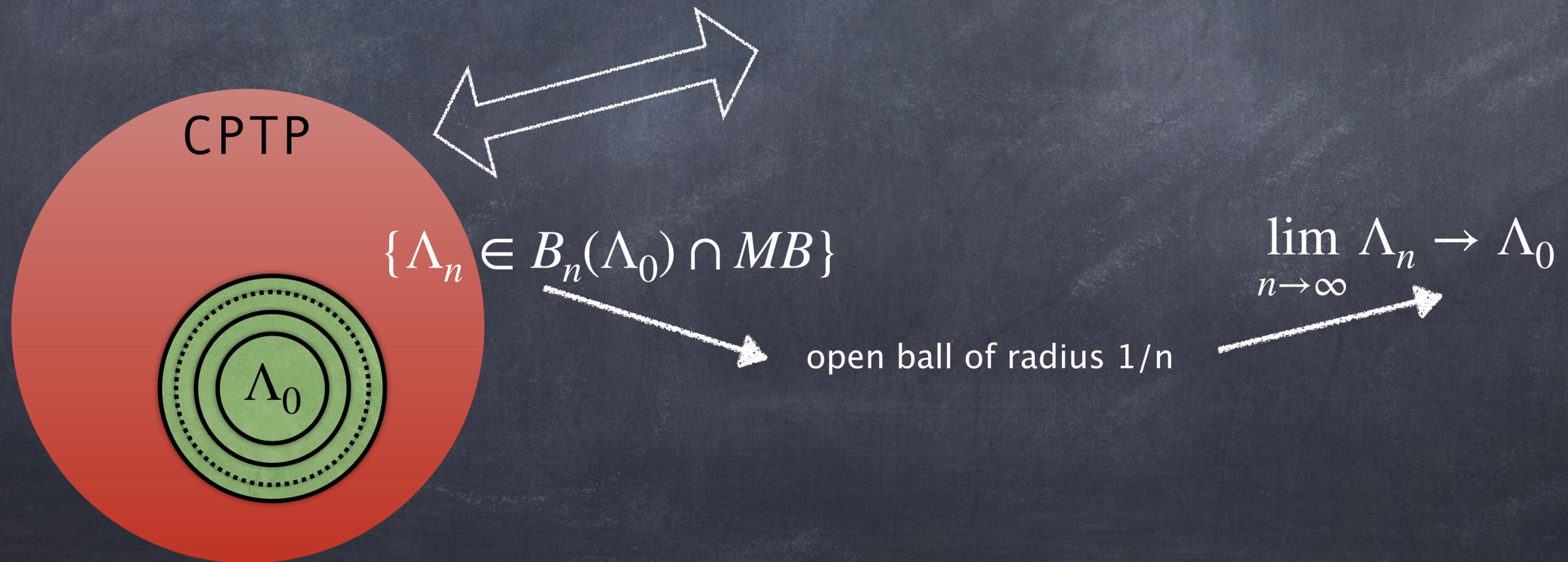
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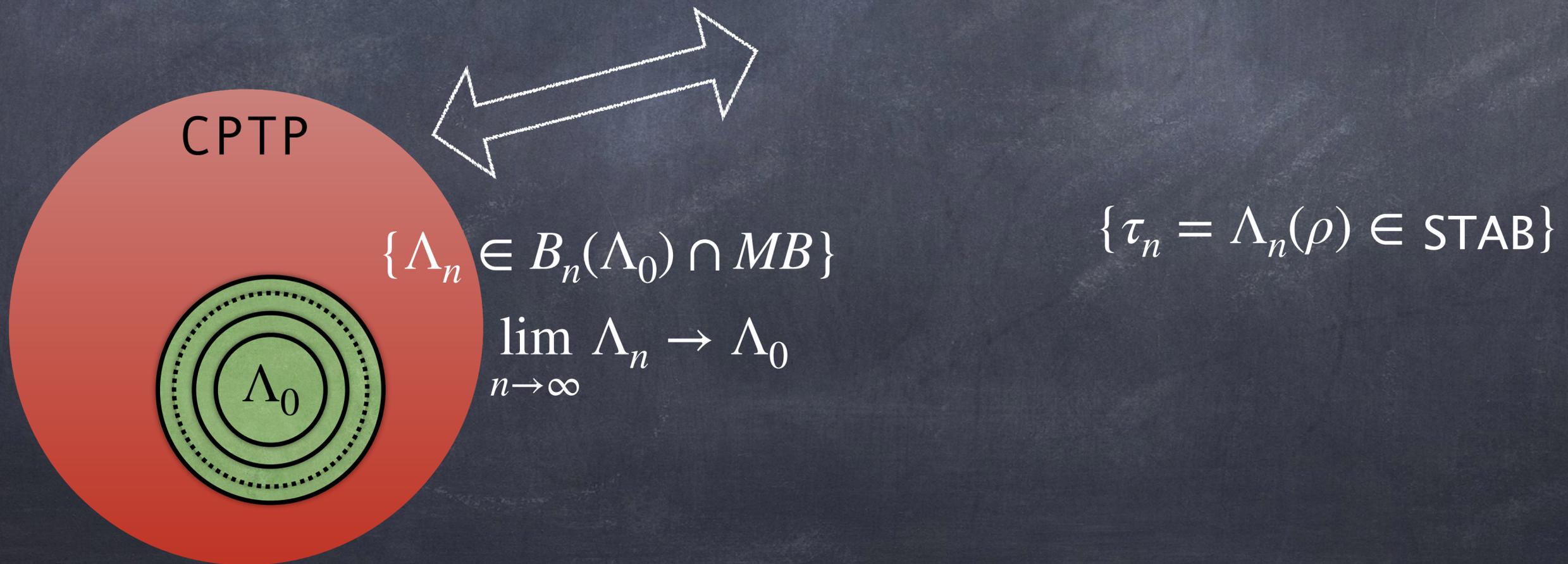
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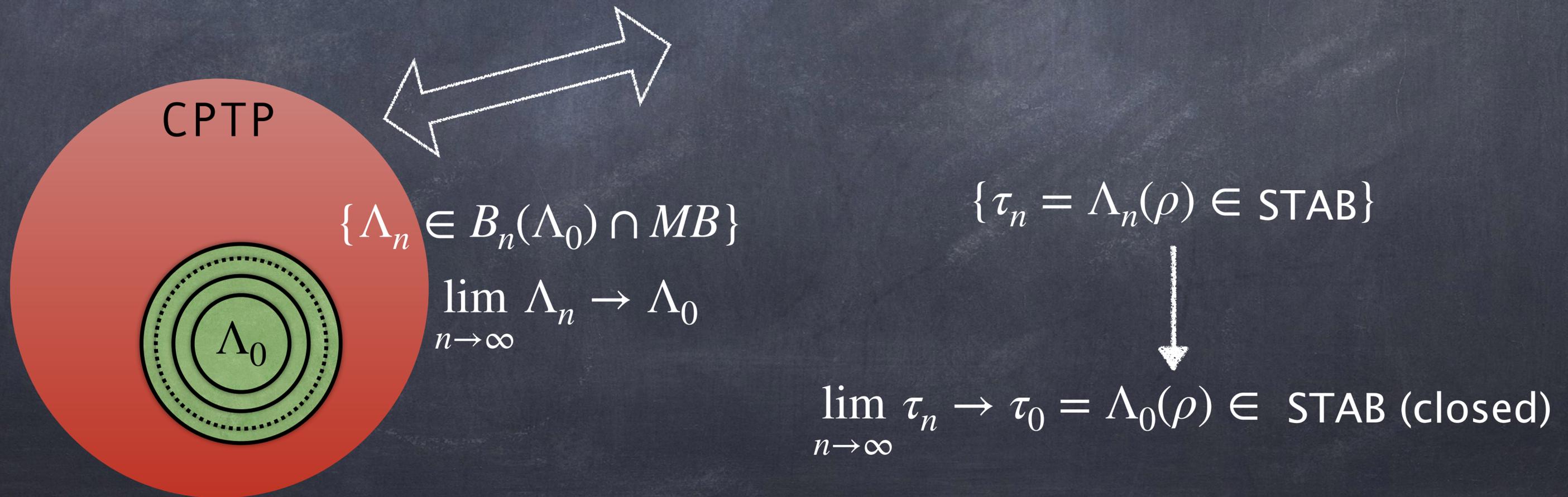
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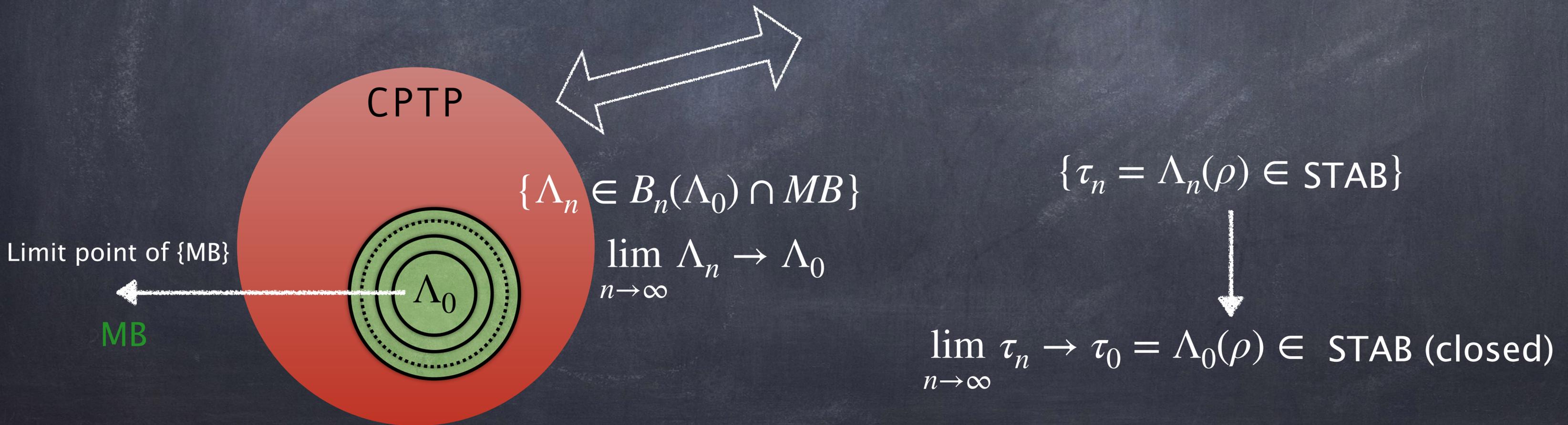
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Properties

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Properties

Extreme points of $\{\text{MB}\}$ contain measure-preapre channels preparing non-orthogonal stabilizer states

$$\sum_k |\eta_k\rangle\langle\eta_k| \langle e_k|\rho|e_k\rangle : \langle e_j|e_k\rangle = \delta_{jk} \ \& \ \langle\eta_j|\eta_k\rangle \neq 0$$

Extreme CQ channels preparing non-orthogonal states are extreme CPTP maps

Bucket List

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Qubit channels - geometric analysis

Lin. Alg. Appl. 347 159 (2002)

$$\Lambda = U_{post} \circ \Lambda_C \circ U_{pre}$$

Qubit channels - geometric analysis

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→ Gives valid state - Can ignore this

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Gives valid state - Can ignore this

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ t_1 & \lambda_1 & 0 & 0 \\ t_2 & 0 & \lambda_2 & 0 \\ t_3 & 0 & 0 & \lambda_3 \end{pmatrix}$$

Qubit channels - geometric analysis

Lin. Alg. Appl. 347 159 (2002)

$$\Lambda = U_{post} \circ \Lambda_C \circ U_{pre}$$

Gives valid state - Can ignore this

$$\begin{pmatrix} \cos \frac{\theta}{2} e^{i(2\pi - \phi - \psi)/2} & i \sin \frac{\theta}{2} e^{-i(\phi - \psi)/2} \\ i \sin \frac{\theta}{2} e^{i(\phi - \psi)/2} & \cos \frac{\theta}{2} e^{-i(2\pi - \phi - \psi)/2} \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ t_1 & \lambda_1 & 0 & 0 \\ t_2 & 0 & \lambda_2 & 0 \\ t_3 & 0 & 0 & \lambda_3 \end{pmatrix}$$

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$$\rho(\{m_i\}) = \frac{1}{2}(\mathbb{I}_2 + \sum_i m_i \sigma_i)$$

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$$\rho(\{m_i\}) = \frac{1}{2}(\mathbb{1}_2 + \sum_i m_i \sigma_i) \xrightarrow{\Lambda_C} \rho(\{m'_i\}) = \frac{1}{2}(\mathbb{1}_2 + \sum_i m'_i \sigma_i)$$

$$m'_i(m_i, \lambda_i, t_i) = \lambda_i m_i + t_i$$

Qubit channels - geometric analysis

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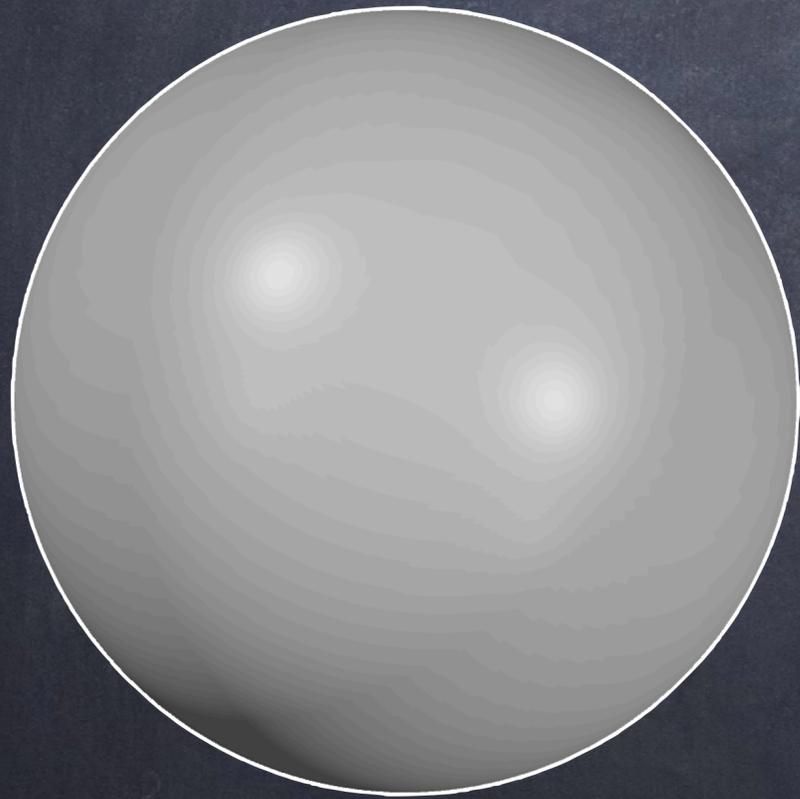
$$m'_i(m_i, \lambda_i, t_i) = \lambda_i m_i + t_i$$

$$m''_i = f(m'_i, \theta, \phi, \psi)$$

Qubit channels - geometric analysis

Lin. Alg. Appl. 347 159 (2002)

$$\Lambda = U_{post} \circ \Lambda_C \circ U_{pre}$$

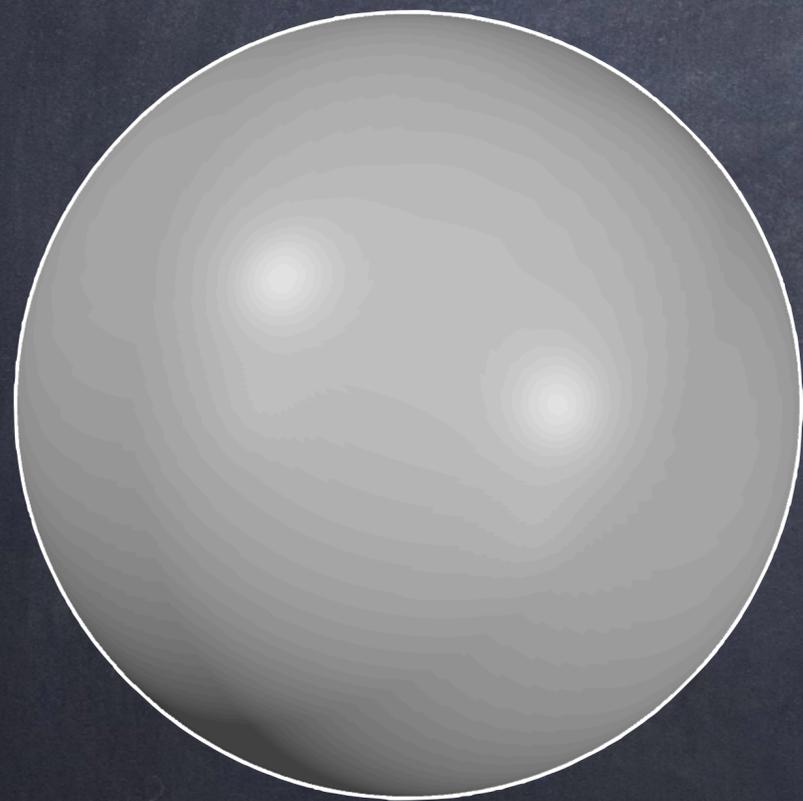


Sphere

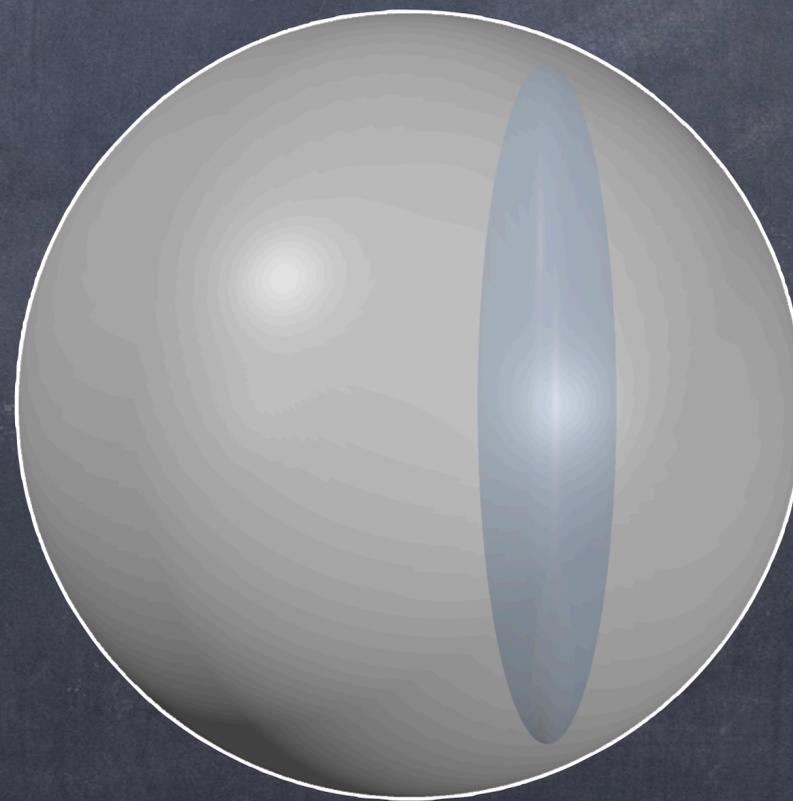
Qubit channels - geometric analysis

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Sphere

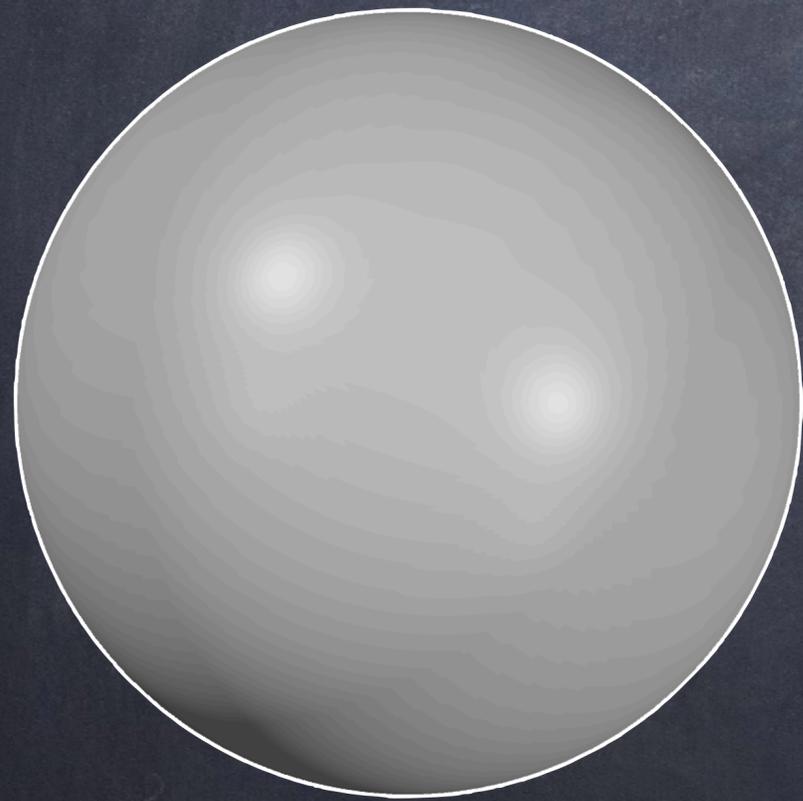


Shifted ellipsoid

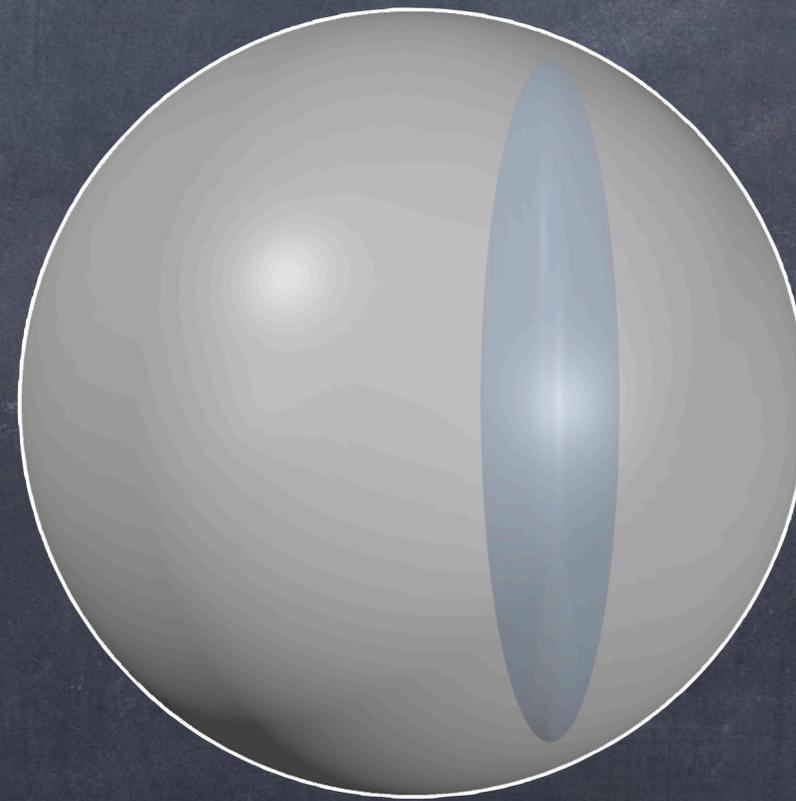
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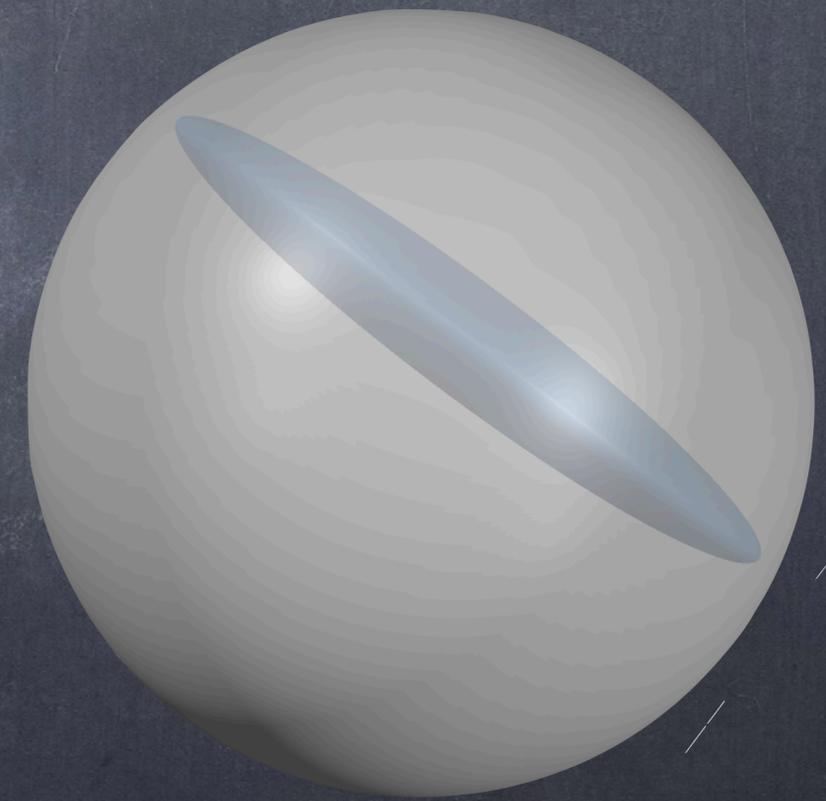
$$\Lambda = U_{post} \circ \Lambda_C \circ U_{pre}$$



Sphere



Shifted ellipsoid



Rotated shifted ellipsoid

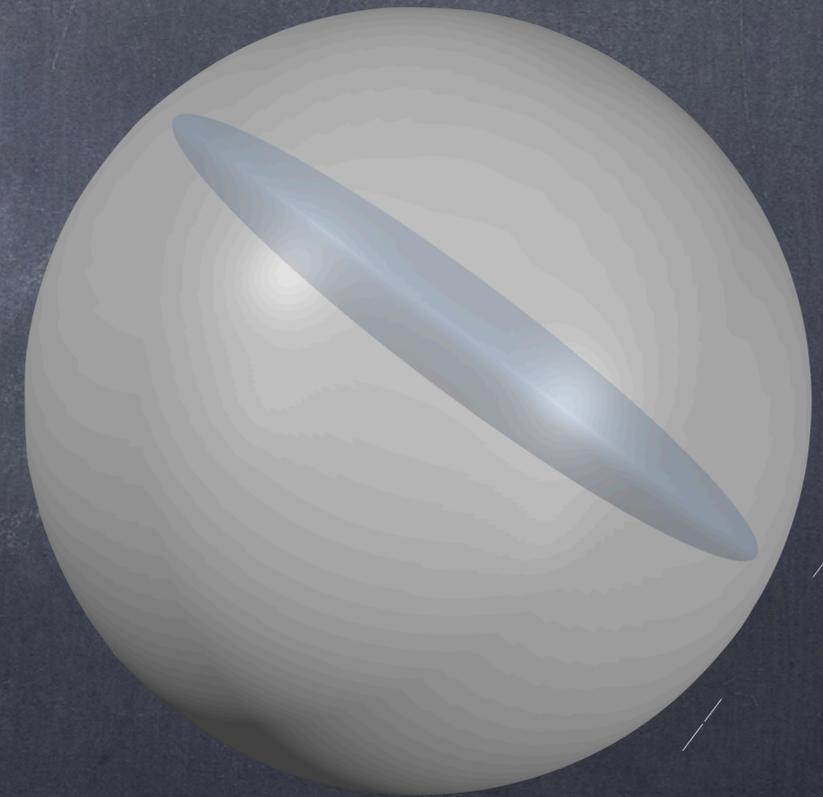
Qubit channels - geometric analysis

$$\Lambda = U_{post} \circ \Lambda_C \circ U_{pre}$$

$$\mu_1 = \frac{\sin \psi \left(m_3'' \sin \theta + \cos \theta (m_2'' \cos \phi - m_1'' \sin \phi) \right) \cos \psi (m_1'' \cos \phi + m_2'' \sin \phi)}{\lambda_1} - \frac{t_1}{\lambda_1}$$

$$\mu_2 = \frac{m_3'' \cos \psi \sin \theta - \sin \psi (m_1'' \cos \phi + m_2'' \sin \phi) \cos \theta \cos \psi (m_2'' \cos \phi - m_1'' \sin \phi)}{\lambda_2} - \frac{t_2}{\lambda_2},$$

$$\mu_3 = \frac{\cos \theta (m_3'' - m_2'' \cos \phi \tan \theta + m_1'' \sin \phi \tan \theta)}{\lambda_3} - \frac{t_3}{\lambda_3}$$

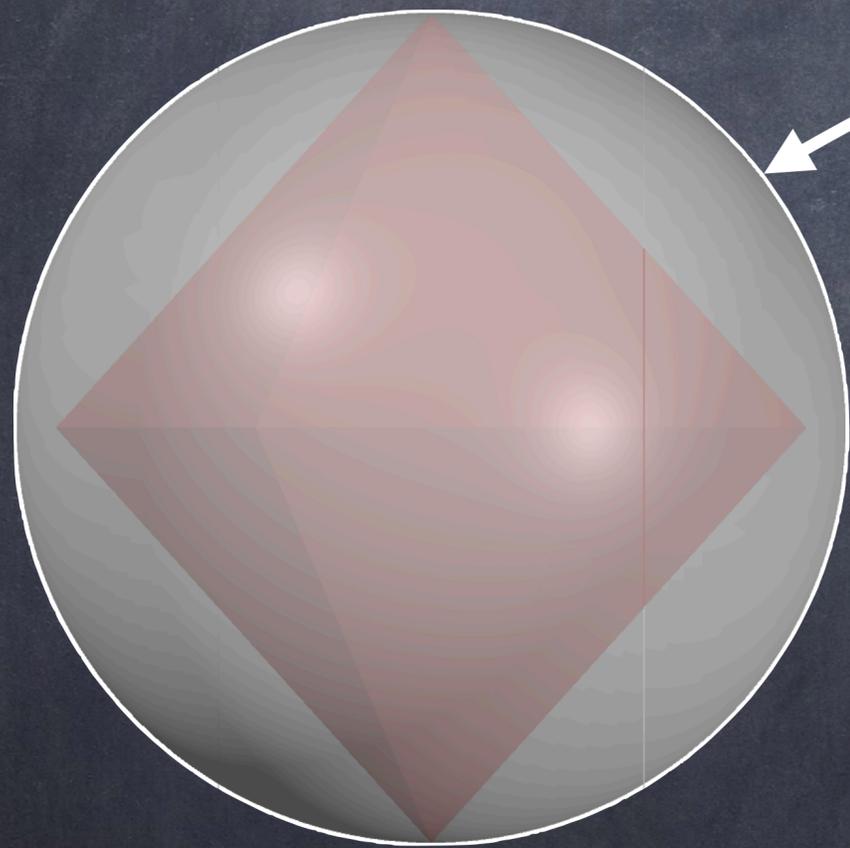


Rotated shifted ellipsoid

Bucket List

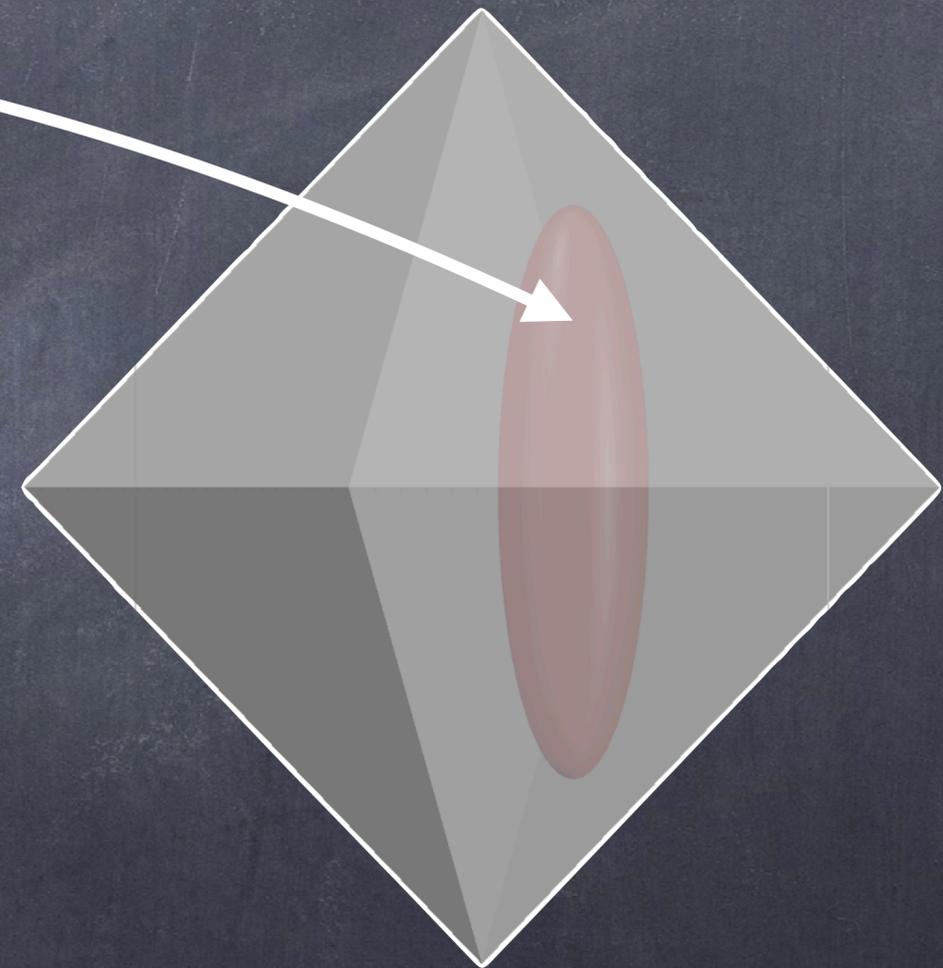
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Qubit MB channels - geometric analysis



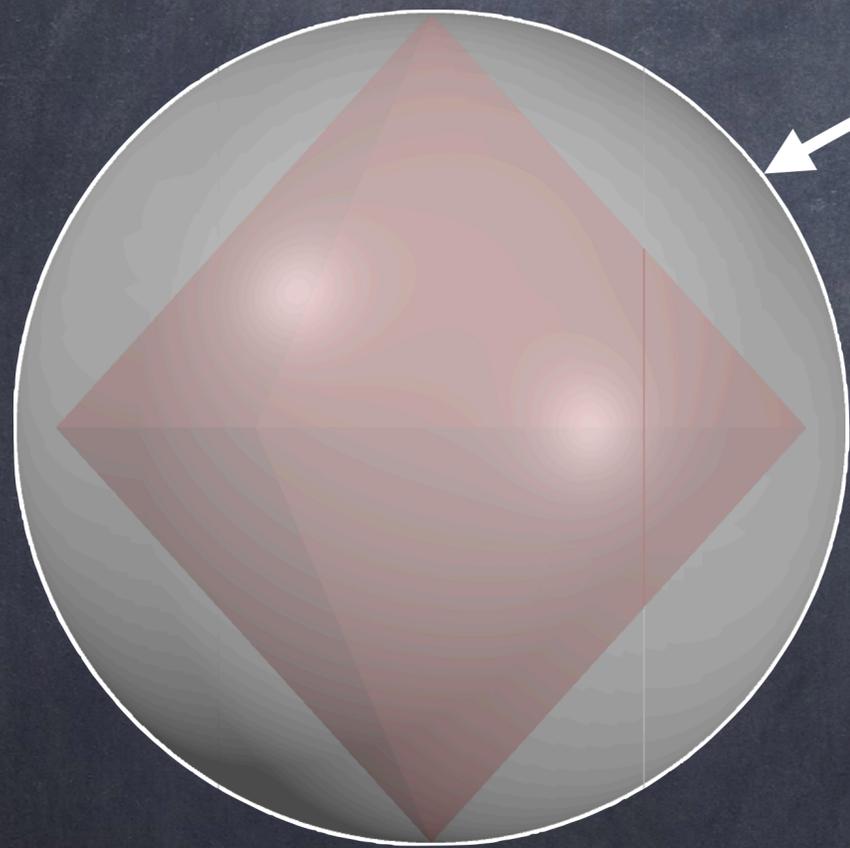
Stabilizer polytope
(useless states for UQC)

$$\Lambda_{MB}(\rho) \in STAB$$



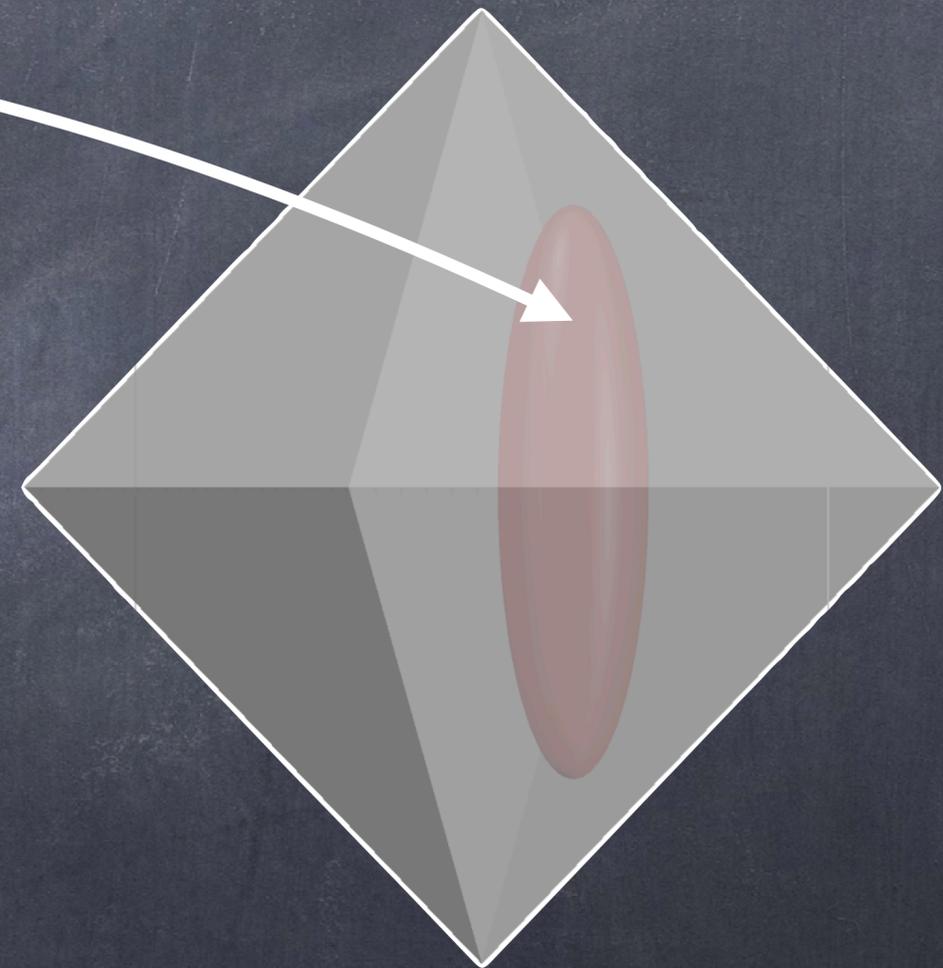
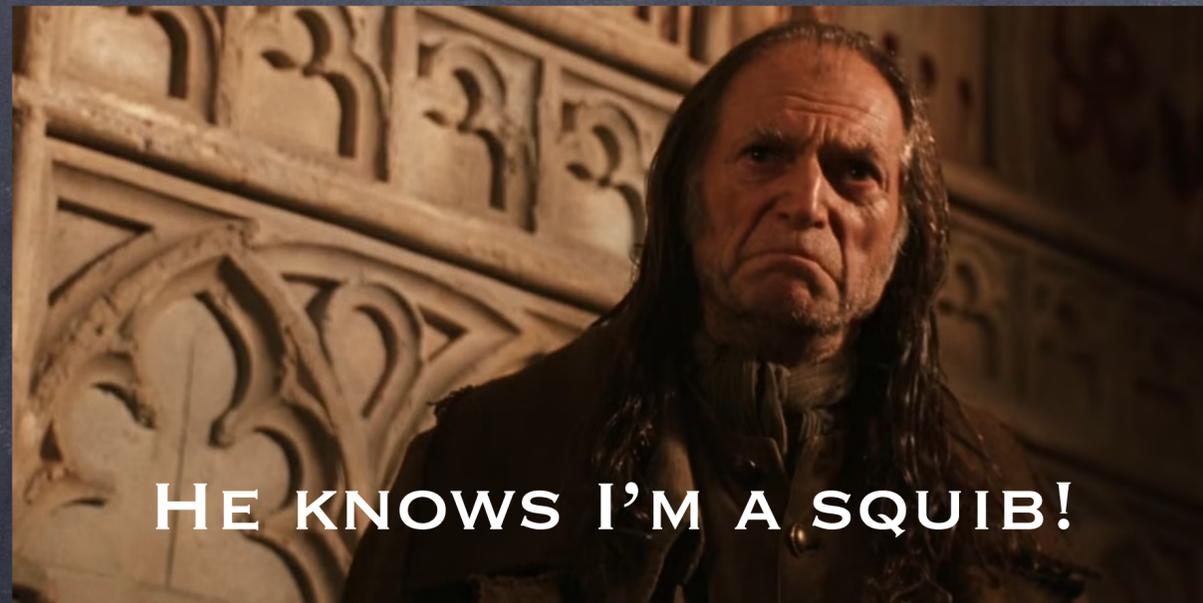
Rotated shifted ellipsoid
within stabilizer polytope

Qubit MB channels - geometric analysis



Stabilizer polytope
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Rotated shifted ellipsoid
within stabilizer polytope

Condition for MB

Simultaneously solve the rotated shifted ellipsoid and the stabilizer polytope equations

Condition for MB

Simultaneously solve the rotated shifted ellipsoid and the stabilizer polytope equations

 Solve ellipsoid with $m_1'' + m_2'' + m_3'' = 1$

$$m_j'' = f_j(m_1'', \{\lambda_k, t_k\}, \theta, \phi, \psi) \pm \sqrt{\alpha m_1''^2 + \beta m_1'' + \gamma}$$

Condition for MB

Simultaneously solve the rotated shifted ellipsoid and the stabilizer polytope equations



Magic broken iff ellipsoid entirely within the polytope – finite or no simultaneous solutions

$$\alpha m_1'''^2 + \beta m_1'' + \gamma \leq 0$$

Condition for MB

Simultaneously solve the rotated shifted ellipsoid and the stabilizer polytope equations



Magic broken iff ellipsoid entirely within the polytope – finite or no simultaneous solutions

$$\alpha m_1''^2 + \beta m_1'' + \gamma \leq 0$$



$$\begin{cases} \beta^2 - 4\alpha\gamma \leq 0 & \text{if } \alpha < 0 \text{ and } \left| \frac{\beta}{4\alpha} \right| \leq 1 \\ \alpha \pm \beta + \gamma \leq 0 & \text{otherwise.} \end{cases}$$

Necessary and sufficient

Condition for MB

Simultaneously solve the rotated shifted ellipsoid and the stabilizer polytope equations

$$\begin{cases} \beta^2 - 4\alpha\gamma \leq 0 & \text{if } \alpha < 0 \text{ and } \left| \frac{\beta}{4\alpha} \right| \leq 1 \\ \alpha \pm \beta + \gamma \leq 0 & \text{otherwise.} \end{cases}$$



Repeat for all polytope faces

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- Define magic-breaking channels (MB)
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Classes of MB channels



Strictly magic-breaking

Classes of MB channels



Strictly magic-breaking



Final ellipsoid inside largest sphere within stabilizer polytope

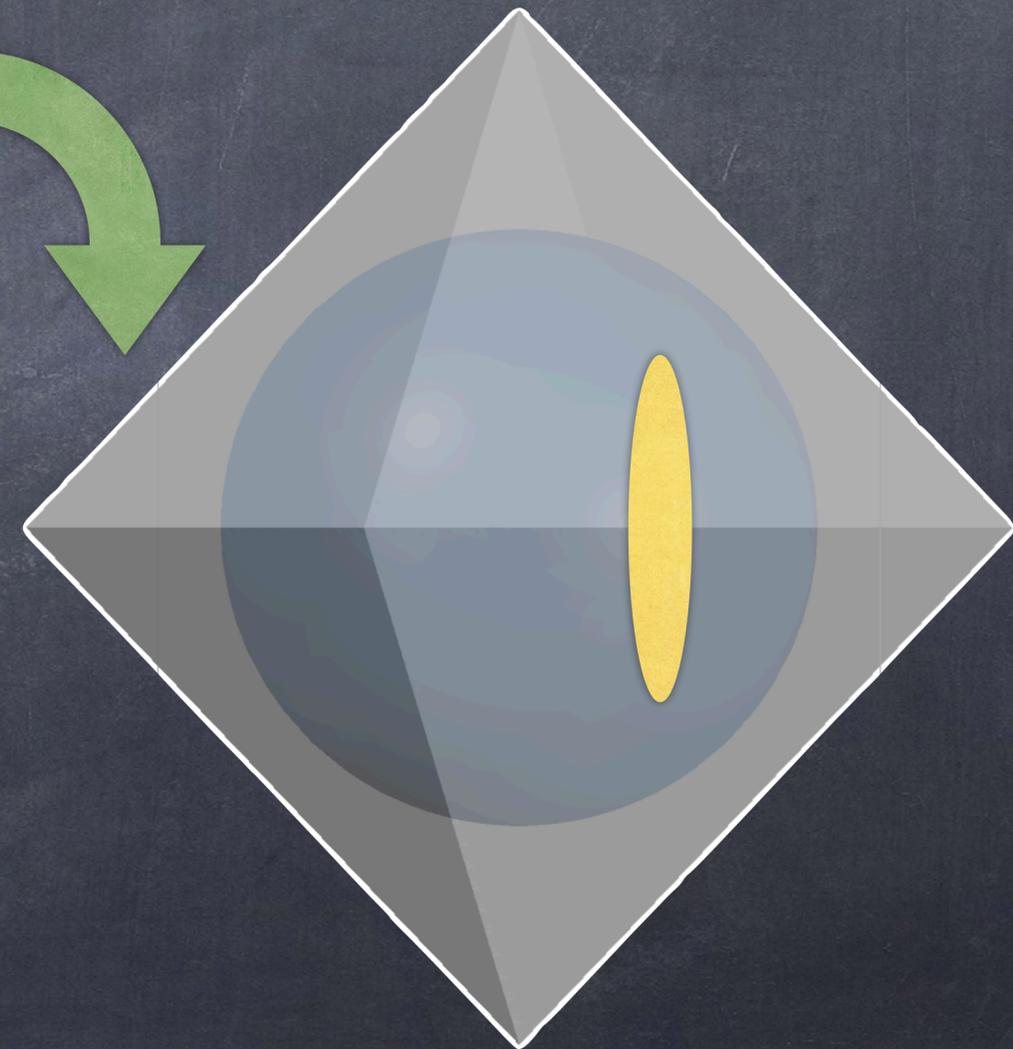
Classes of MB channels



Strictly magic-breaking



Final ellipsoid inside largest sphere within stabilizer polytope



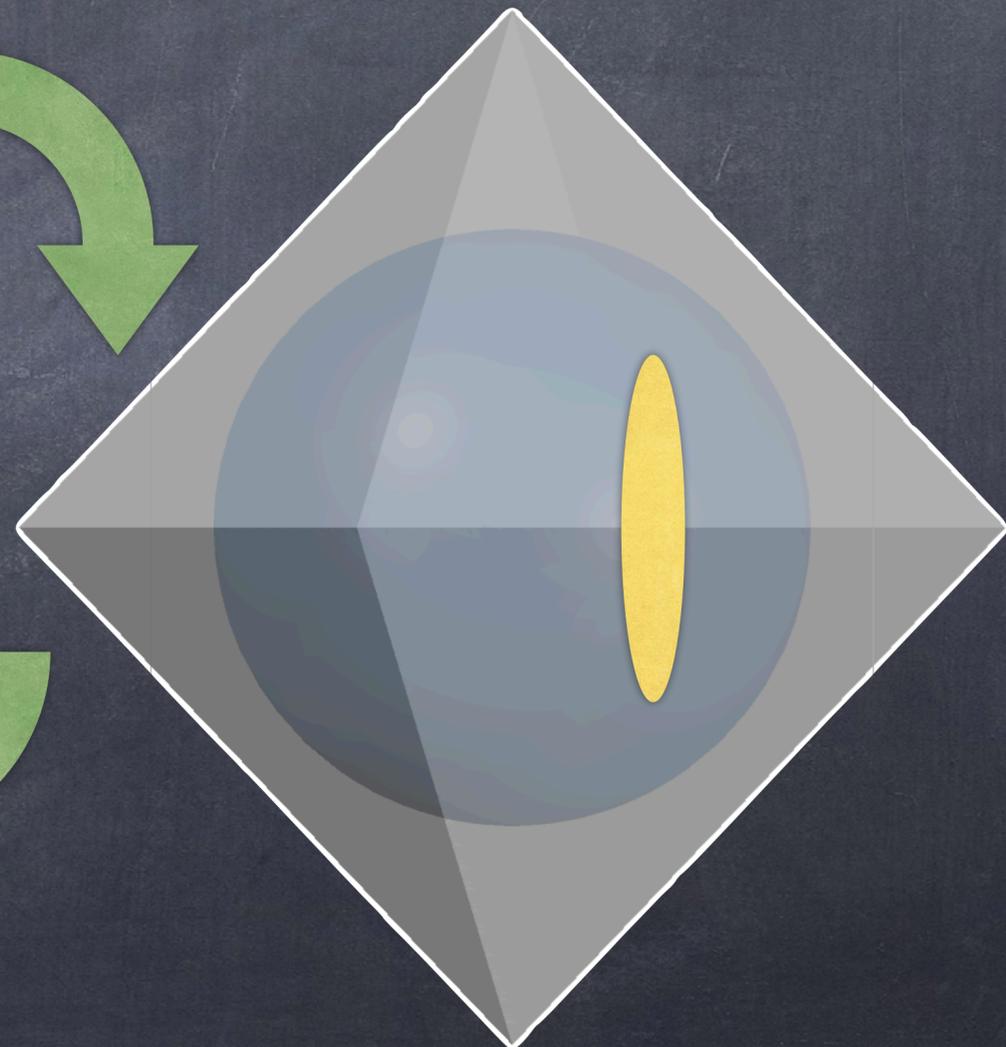
Classes of MB channels



Strictly magic-breaking



Final ellipsoid inside largest sphere within stabilizer polytope



Sufficient


$$|t| + |\lambda_i| \leq \frac{1}{\sqrt{3}} \quad \forall i : |t| = \sqrt{\sum_k t_k^2}$$



Classes of MB channels



Channels with Clifford post-processing

Classes of MB channels



Channels with Clifford post-processing



Final ellipsoid within stabilizer polytope

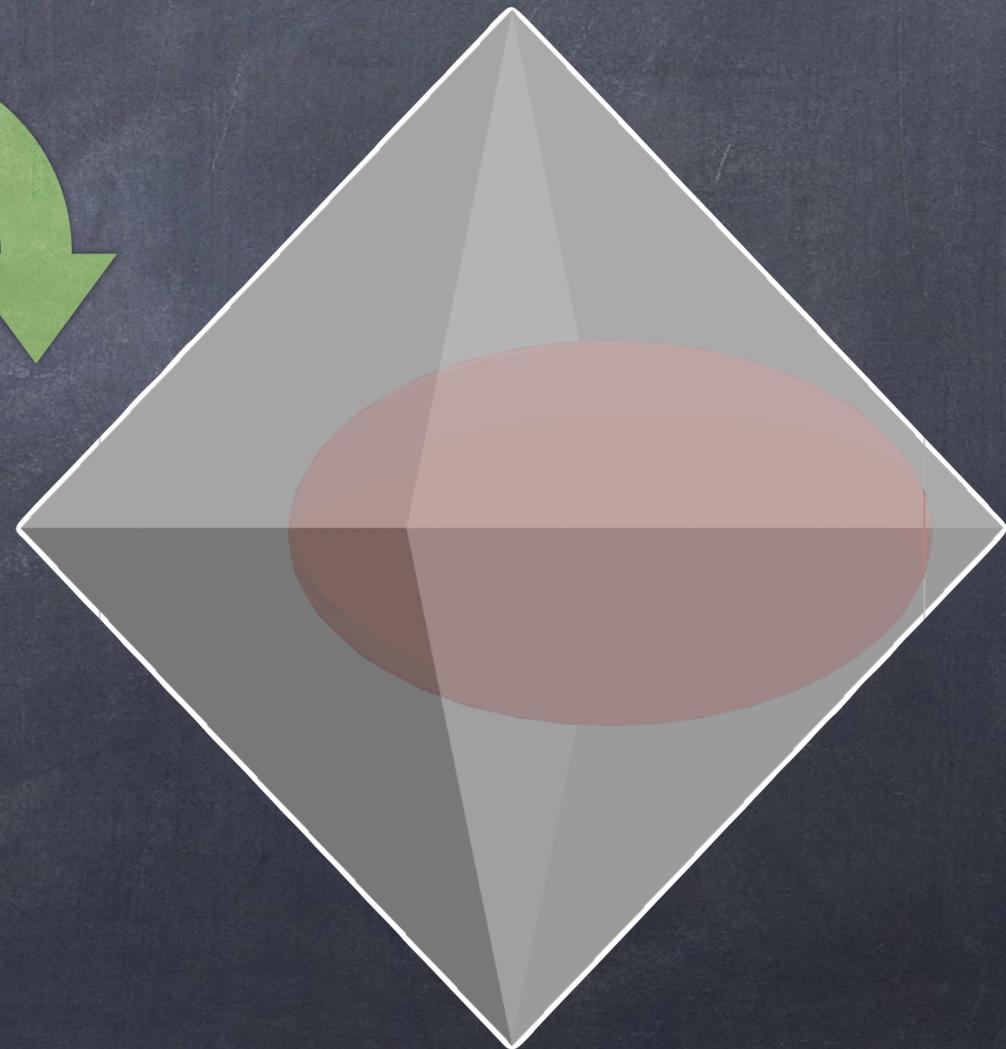
Classes of MB channels



Channels with Clifford post-processing



Final ellipsoid within stabilizer polytope



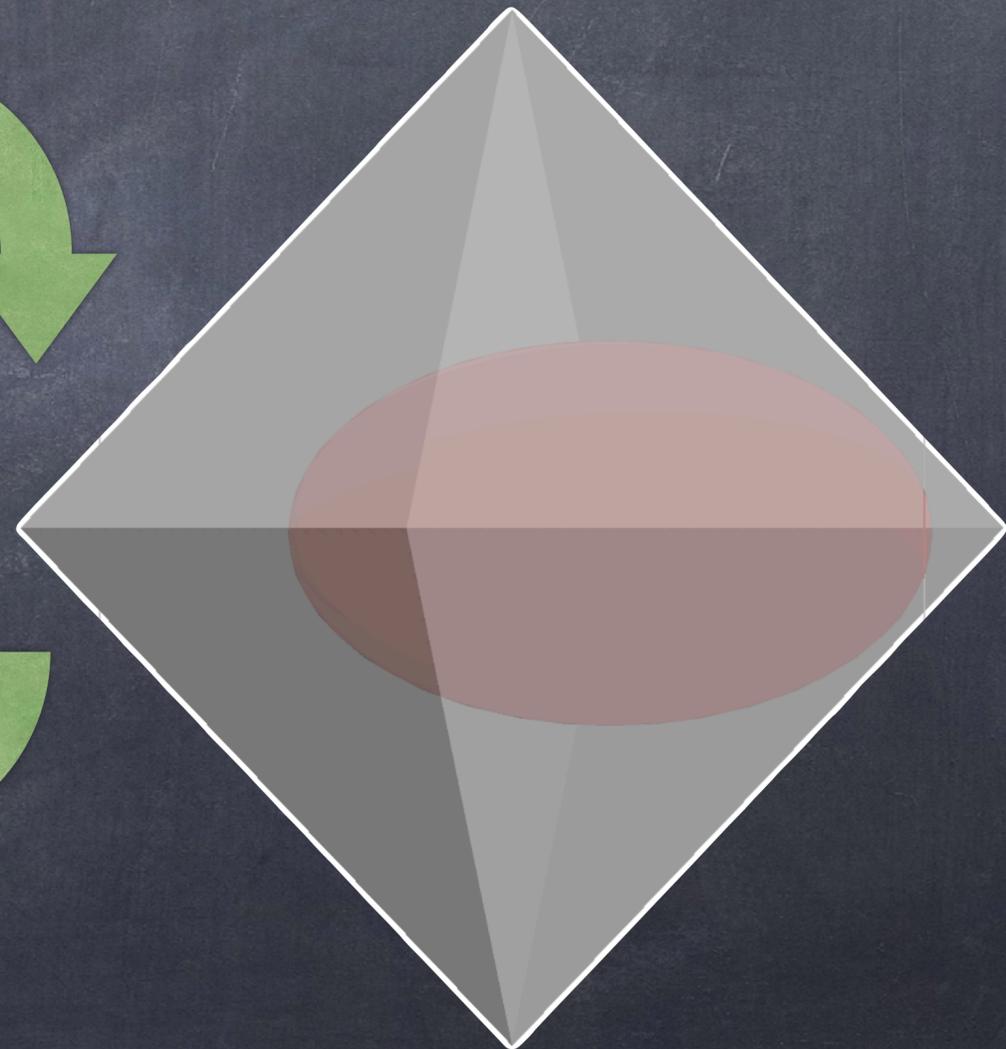
Classes of MB channels



Channels with Clifford post-processing



Final ellipsoid within stabilizer polytope



Necessary and sufficient

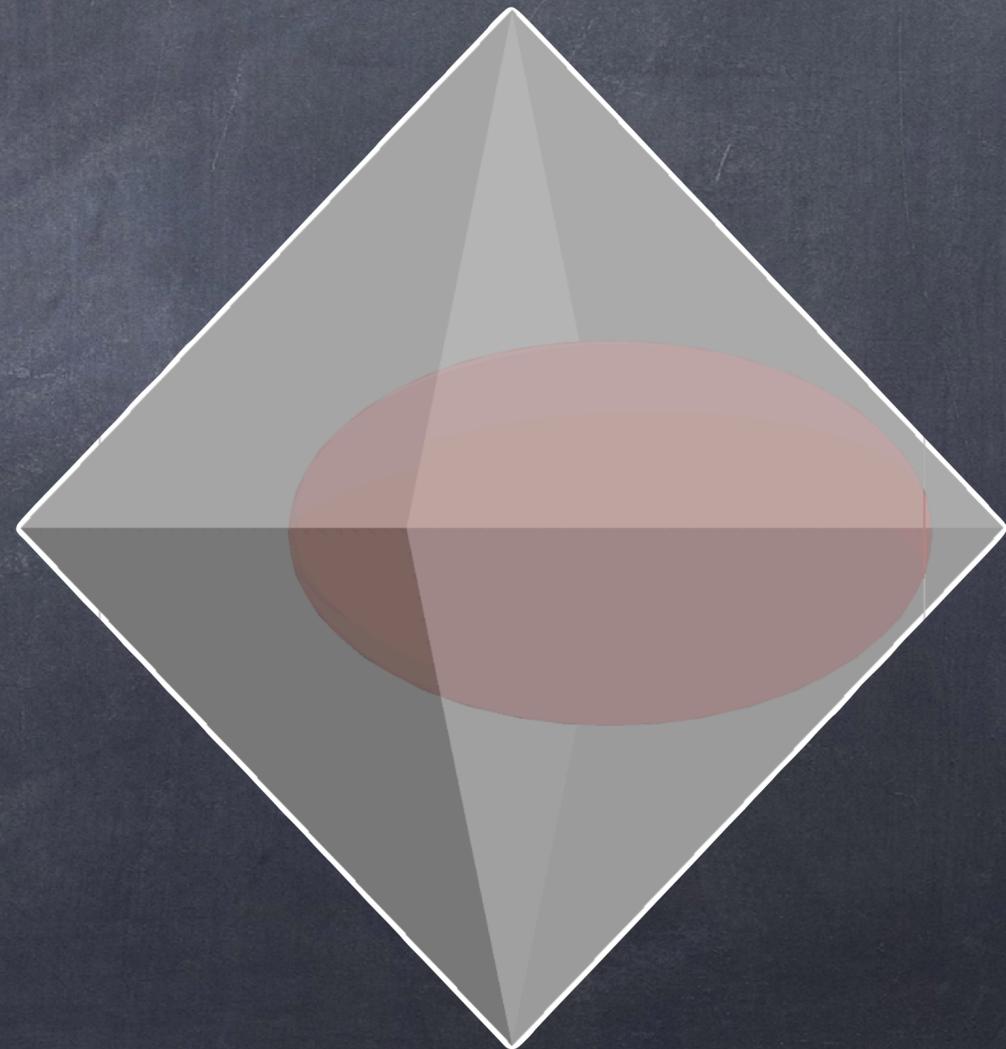

$$\sum_i \lambda_i^2 \leq \left(1 - \sum_i |t_k|\right)^2$$



Classes of MB channels



Channels with Clifford post-processing – Pauli channels and unital EBT



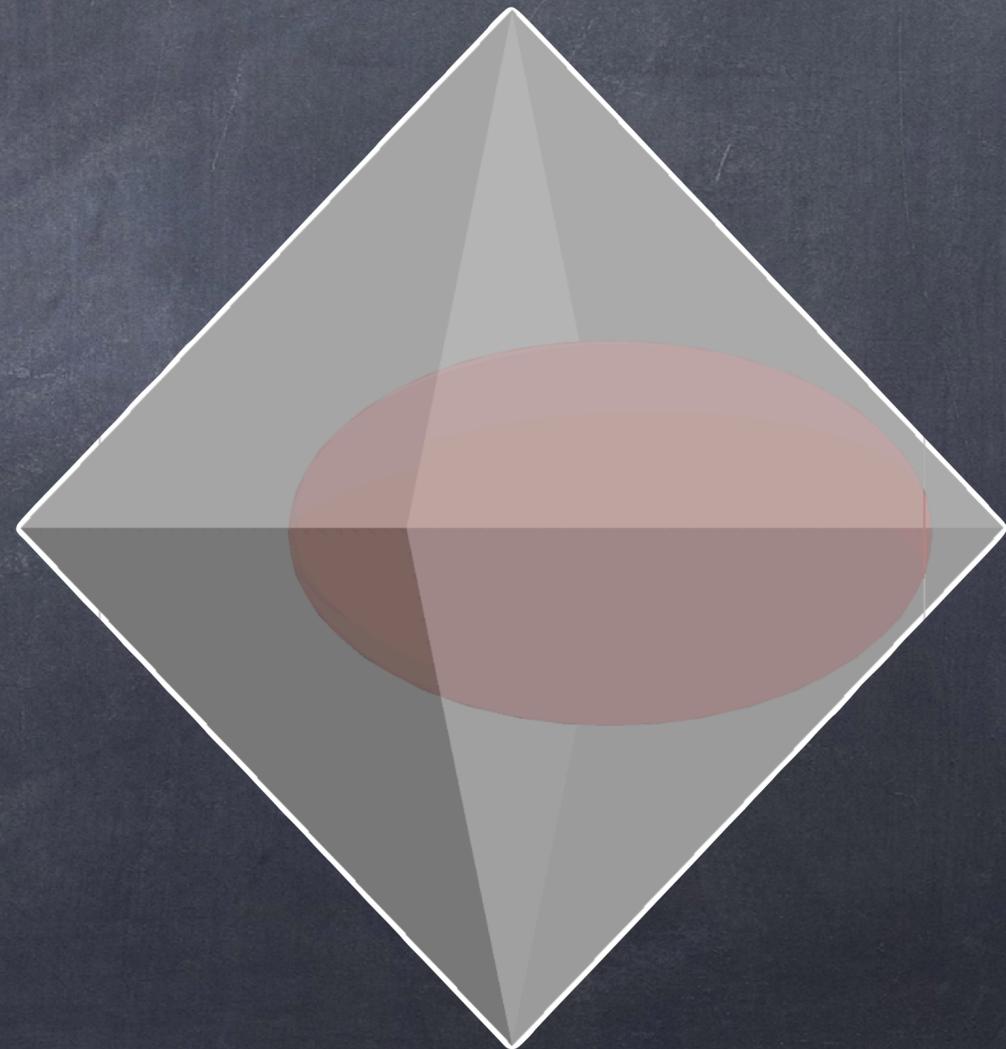
Classes of MB channels



Channels with Clifford post-processing – Pauli channels and unital EBT

Dephasing


$$\rho \rightarrow (1 - p/2)\rho + \frac{p}{2}\sigma_z\rho\sigma_z : p = 1$$



Classes of MB channels

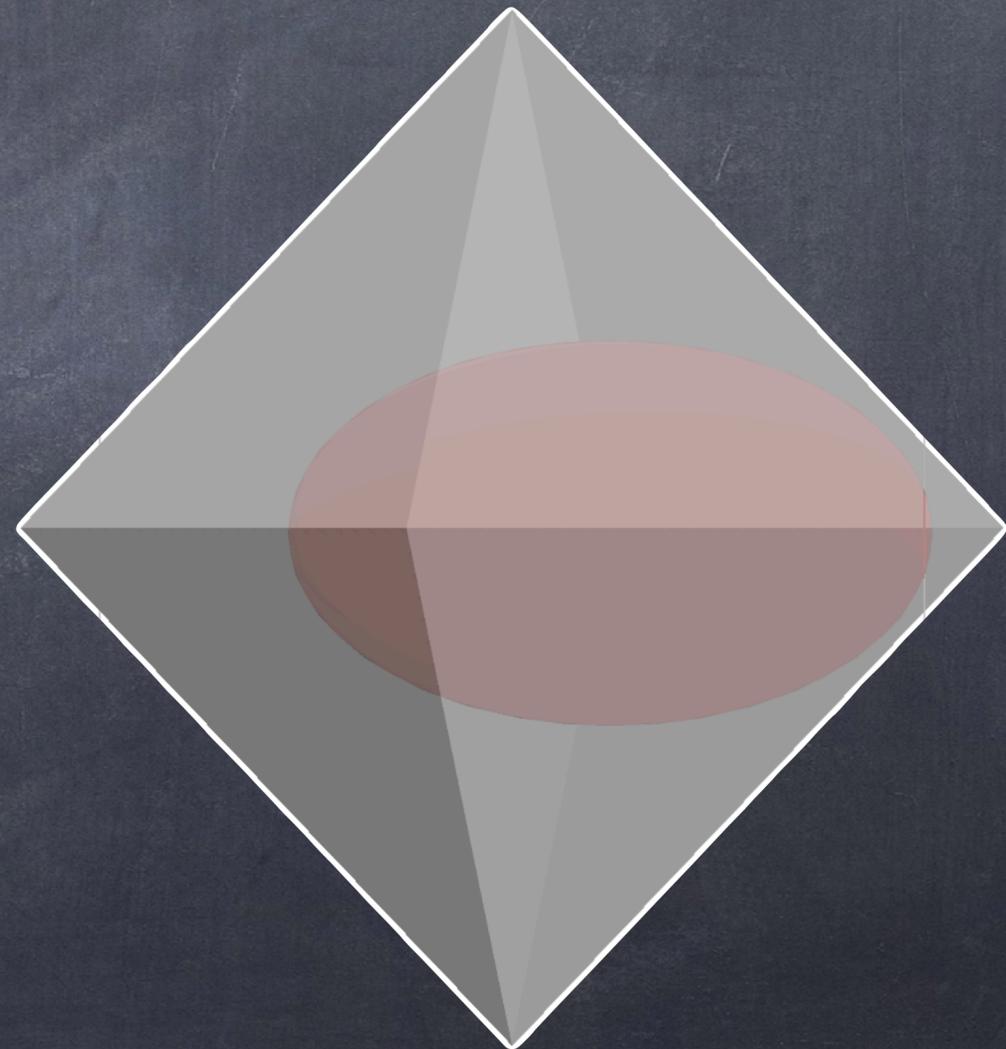


Channels with Clifford post-processing – Pauli channels and unital EBT

$$\rho \rightarrow (1 - p/2)\rho + \frac{p}{2}\sigma_z\rho\sigma_z : p = 1$$

Depolarising

$$\rho \rightarrow p\mathbb{I}/2 + (1 - p)\rho : p \geq 1 - 1/\sqrt{3}$$



Classes of MB channels

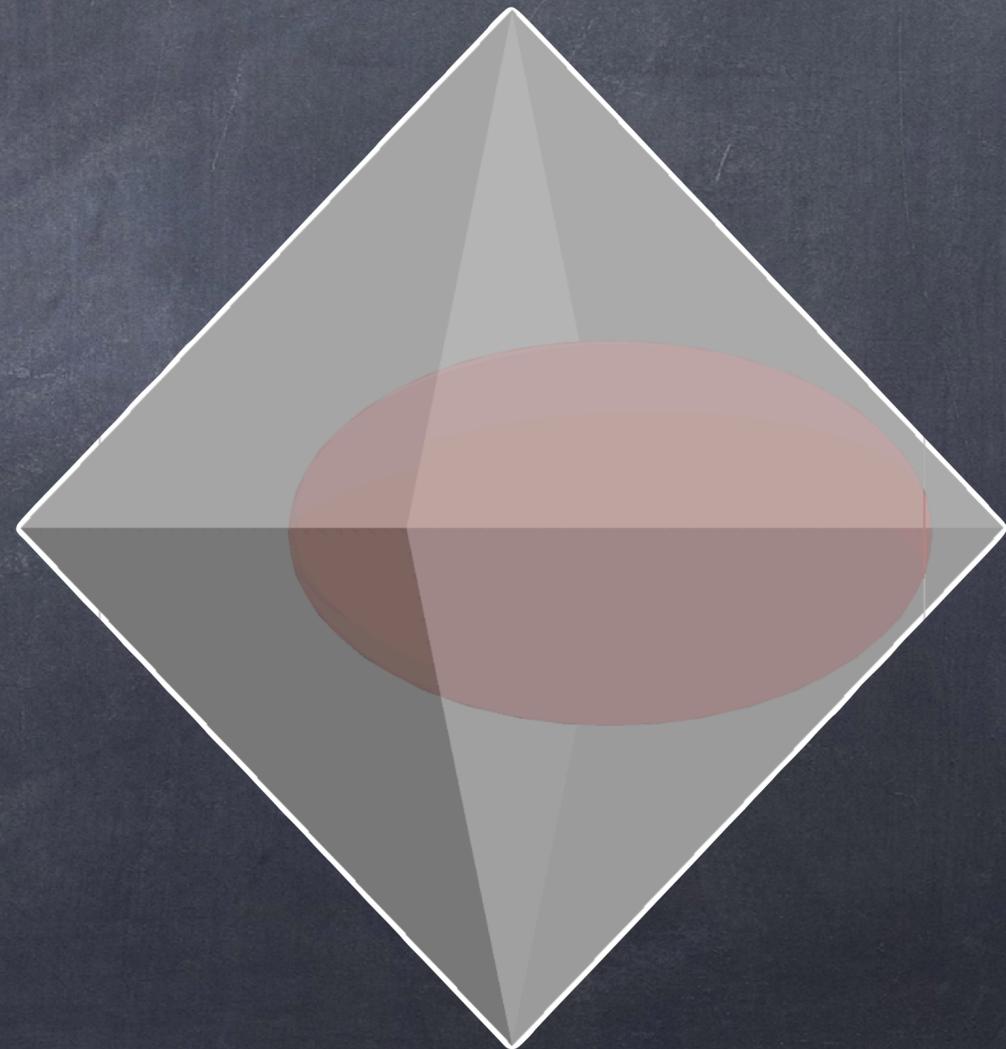


Channels with Clifford post-processing – Pauli channels and unital EBT

$$\rho \rightarrow (1 - p/2)\rho + \frac{p}{2}\sigma_z\rho\sigma_z : p = 1$$

Unital EBT

$$\sum_i |\lambda_i| \leq 1 \implies \sum_i \lambda_i^2 \leq 1 \quad (|\lambda_i| \leq 1)$$



Classes of MB channels



T-distillability breaking channels ($\mathcal{R}_\rho > 3/\sqrt{7}$: T-distillable)

Classes of MB channels



T-distillability breaking channels ($\mathcal{R}_\rho > 3/\sqrt{7}$: T-distillable)

Solve for no simultaneous solution of final ellipsoid and $\sum_i |m_i| = 3/\sqrt{7}$

Clifford post-processing


$$\sum_i \lambda_i^2 \leq \left(\frac{3}{\sqrt{7}} - \sum_i |t_k| \right)^2$$

Classes of MB channels



T-distillability breaking channels ($\mathcal{R}_\rho > 3/\sqrt{7}$: T-distillable)

Solve for no simultaneous solution of final ellipsoid and $\sum_i |m_i| = 3/\sqrt{7}$

Clifford post-processing

$$\sum_i \lambda_i^2 \leq \left(\frac{3}{\sqrt{7}} - \sum_i |t_k| \right)^2$$

Depolarising

$$p \geq 1 - \sqrt{3/7}$$

Dephasing

$$p \geq 0.622$$

Bucket List

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Multiqubit MB channels

Necessary



$\bigotimes_{i=1}^N \Lambda_i \in MB$ only if $\Lambda_i \in MB \forall i$

Multiqubit MB channels

Necessary



$\bigotimes_{i=1}^N \Lambda_i \in MB$ only if $\Lambda_i \in MB \forall i$

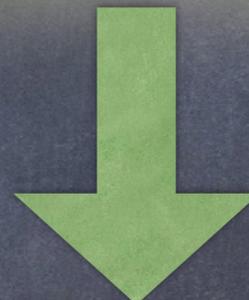


$$\text{Tr}_{\bar{j}} \left[\underbrace{\bigotimes_{i=1}^2 \Lambda_i(\rho_{1,2,\dots,j-1,j,j+1,\dots,N})}_{\text{}} \right] = \Lambda_j(\rho_j) \notin STAB$$

Multiqubit MB channels

Necessary


$$\bigotimes_{i=1}^N \Lambda_i \in MB \text{ only if } \Lambda_i \in MB \forall i$$



$$\text{Tr}_{\bar{j}} \left[\underbrace{\bigotimes_{i=1}^2 \Lambda_i(\rho_{1,2,\dots,j-1,j,j+1,\dots,N})}_{STAB} \right] = \Lambda_j(\rho_j) \notin STAB$$

STAB

Resource generation through
free operation!!

Multiqubit MB channels

Insufficient



$\bigotimes_{i=1}^N \Lambda_i \in MB$ only if $\Lambda_i \in MB \forall i$

$$\mathcal{R}(\Lambda^{\otimes 2} |\eta\rangle) = 1.0212 : \mathcal{R}(|\eta\rangle) = 1.834$$

Multiqubit MB channels

Insufficient



$\bigotimes_{i=1}^N \Lambda_i \in MB$ only if $\Lambda_i \in MB \forall i$

$$\mathcal{R}(\Lambda^{\otimes 2} |\eta\rangle) = 1.0212 : \mathcal{R}(|\eta\rangle) = 1.834$$



$$\Lambda_C = (\lambda_1 = -0.9, \lambda_2 = -0.3, \lambda_3 = 0.2, t_i = 0); U_{post} = \mathbb{I}$$

Multiqubit MB channels

Insufficient


$$\bigotimes_{i=1}^N \Lambda_i \in MB \text{ only if } \Lambda_i \in MB \forall i$$

$$\mathcal{R}(\Lambda^{\otimes 2} |\eta\rangle) = 1.0212 : \mathcal{R}(|\eta\rangle) = 1.834$$

$$\Lambda_C = (\lambda_1 = -0.9, \lambda_2 = -0.3, \lambda_3 = 0.2, t_i = 0); U_{post} = \mathbb{I}$$

$$(-0.482 - i0.648) |00\rangle + (0.015 - 0.022i) |01\rangle + (-0.131 - 0.098i) |10\rangle + (-0.145 - 0.548i) |11\rangle$$

Multiqubit MB channels: Consequences of insufficiency



$$\bigotimes_{j=1}^N \Lambda_j^{MB} \left(\sum_i p_i \rho_i^1 \otimes \dots \otimes \rho_i^N \right) \in STAB$$



Tensor product of MB channels cannot destroy magic present in correlations

Multiqubit MB channels: Consequences of insufficiency



$$\bigotimes_{j=1}^N \Lambda_j^{MB} \left(\sum_i p_i \rho_i^1 \otimes \dots \otimes \rho_i^N \right) \in STAB$$



Tensor product of MB channels cannot destroy magic present in correlations



$$\bigotimes_{i=1}^N \Lambda_i^{MB} \circ \bigotimes_{j=1}^{N-1} \Lambda_j^{EBT} \in MB$$

Multiqubit MB channels: Consequences of insufficiency



Dynamical resource theory of magic preservability : activation of resource preservability

Quantum 4, 244 (2020)

Multiqubit MB channels: Consequences of insufficiency

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 $\otimes_i \rho_i^{STAB} \in STAB$: {STAB} constitutes *absolutely free states*

no resource activation like non-locality

Multiqubit MB channels: Consequences of insufficiency



Dynamical resource theory of magic preservability : activation of resource preservability

Quantum 4, 244 (2020)

no resource activation like non-locality

$\Theta(\mathcal{E}) := \mathcal{P} \circ (\mathcal{E} \otimes \tilde{\Lambda}) \circ \mathcal{Q}$ - superchannel with stabilizer pre- and post-processing

Multiqubit MB channels: Consequences of insufficiency

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$\Theta(\mathcal{E}) := \mathcal{P} \circ (\mathcal{E} \otimes \tilde{\Lambda}) \circ \mathcal{Q}$ – superchannel with stabilizer pre- and post-processing

absolutely magic-breaking channels : $\tilde{\Lambda} \otimes \Lambda^{MB} \in MB$

Multiqubit MB channels: Consequences of insufficiency



Dynamical resource theory of magic preservability : activation of resource preservability

[Quantum 4, 244 \(2020\)](#)

no resource activation like non-locality

absolutely magic-breaking channels



can destroy magic in correlations – detrimental for UQC

Future directions



Devise distance-based dynamical resource monotnes

Future directions



Devise distance-based dynamical resource monotones



Limitations in practical QC applications

distributed QC : links between quantum processors

blind QC : noisy channel between client and server

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