Second Law of Entanglement Manipulation with Entanglement Battery

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Second law and reversibility

Second law of thermodynamics

$\Delta S \geq 0$

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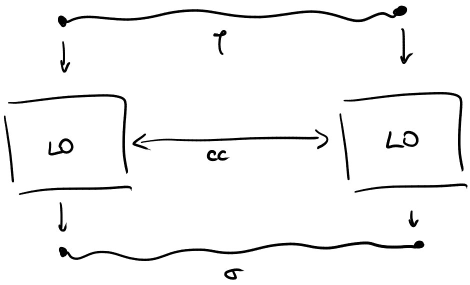
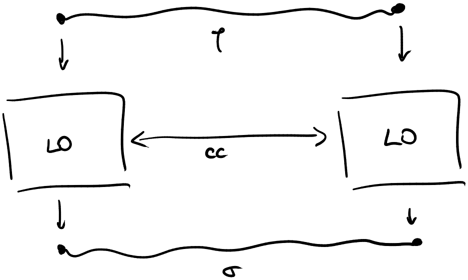
Carnot's theorem

An engine runs at the optimal efficiency iff. it is reversible

Entanglement



Entanglement



1-way LOCC

$\Lambda(\rho) = \sum_{ij} (A_i \otimes B_{ij})\rho(A_i \otimes B_{ij})^\dagger$ with $\sum_i A_i^\dagger A_i = \mathbf{1} = \sum_j B_{ij}^\dagger B_{ij}$.

Separable states

$\rho = \sum_i p_i \rho_A^i \otimes \rho_B^i$.

Reversibility

Asymptotic transformation rate

$R(\rho \to \sigma) = \sup \{ n/m \,|\, \Lambda(\rho^{\otimes m}) \approx_\epsilon \sigma^{\otimes n}, \, \Lambda \text{ LOCC} \}$

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Is entanglement reversible?

No, because of bound entanglement [Horodecki Phys Lett A 1997, Horodecki^3 PRL 1998, Vidal Cirac PRL 2001]

$R(\rho \to \Phi) = 0$, but $R(\Phi \to \rho) < \infty$

Can we make entanglement reversible?

What is known

- PPT entanglement theory is not reversible [Wang Duan PRL 2017]

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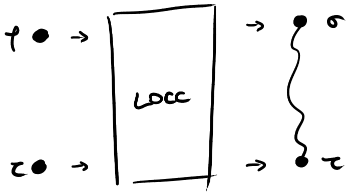
Any physical, non-probabilistic setting?

Clue: catalysis



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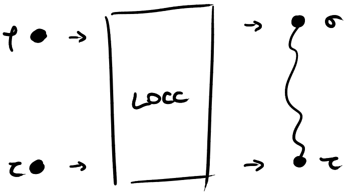

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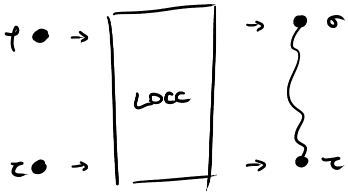

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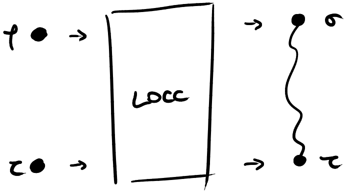

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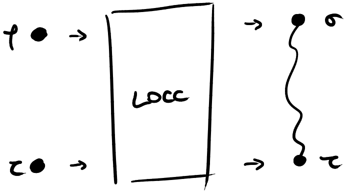

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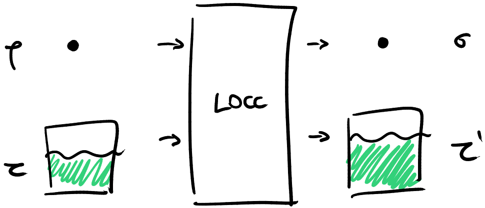
Is it reversible?

No, PPT bound entangled states are still bound entangled catalytically [Lami et al PRA 2024]

Main idea: battery



Fix E.

$\rho \underset{E}{\rightarrow} \sigma$ if $\exists \tau, \Lambda \in$ LOCC s.t.

$$\Lambda(\rho \otimes \tau) = \sigma \otimes \tau',$$

$$E(\tau') \geq E(\tau)$$

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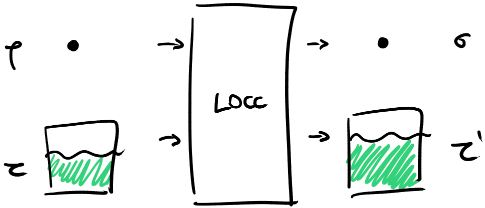

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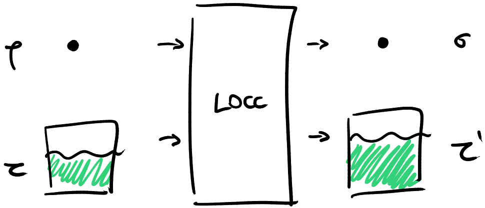

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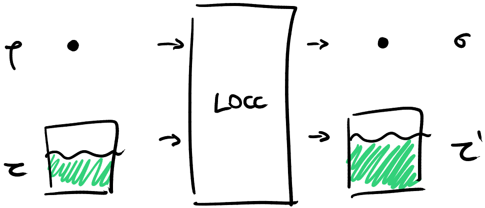

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Squashed entanglement works [Christandl Winter JMP 2004, Li Winter CMP 2014, Alicki Fannes J Phys A 2004], but not uniquely

Results

Theorem 1 (single-copy)

Choose E as a finite and additive entanglement measure.

Then, ρ can be transformed to σ with a battery iff. $E(\rho) \geq E(\sigma)$

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($\Rightarrow$) use properties of E

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Theorem 2 (reversibility)

Choose E as a finite, additive, and asymptotically continuous entanglement measure.

Then

$$R(\rho \underset{E}{\rightarrow} \sigma) = \frac{E(\rho)}{E(\sigma)}$$

How it works

Monotones are “types” of entanglement

$\rho \to \sigma$ iff. $M(\rho) \geq M(\sigma)$ for all monotones M

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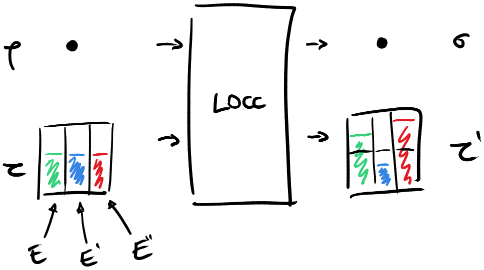

We are supplying other types of entanglement except E

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- What's the value added compared to ARNG?
  Reversibility with LOCC
  Shows reversibility of state transformation is different from maps

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How to get a second law in quantum thermodynamics, in a single shot setting?

Thermodynamics

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Catalytic transformations between energy-incoherent states are governed by free energy [Müller PRX 2018]
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How to extend to coherent states?

Theorem 3

ρ can be transformed to σ with a battery iff. $F(\rho) \geq F(\sigma)$, where

$F(\rho) = k_B T(S(\rho\|\gamma) - \log Z)$

Closing notes

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Thank you!
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