Super-activating quantum memory by entanglement-breaking channels

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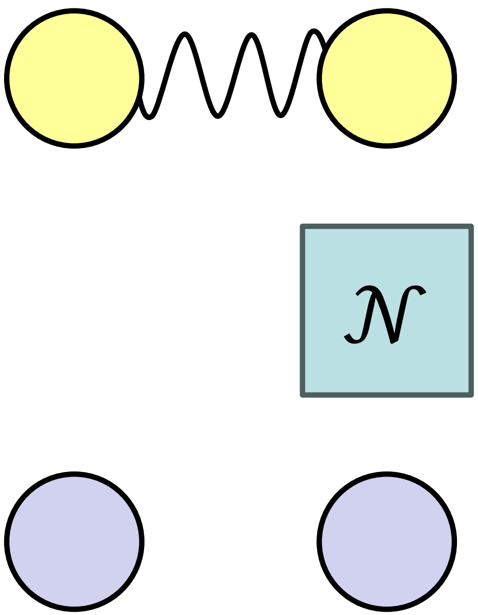
(Jeju, Korea)

Motivation

Are entanglement-breaking (EB) channels useless for maintaining entanglement?



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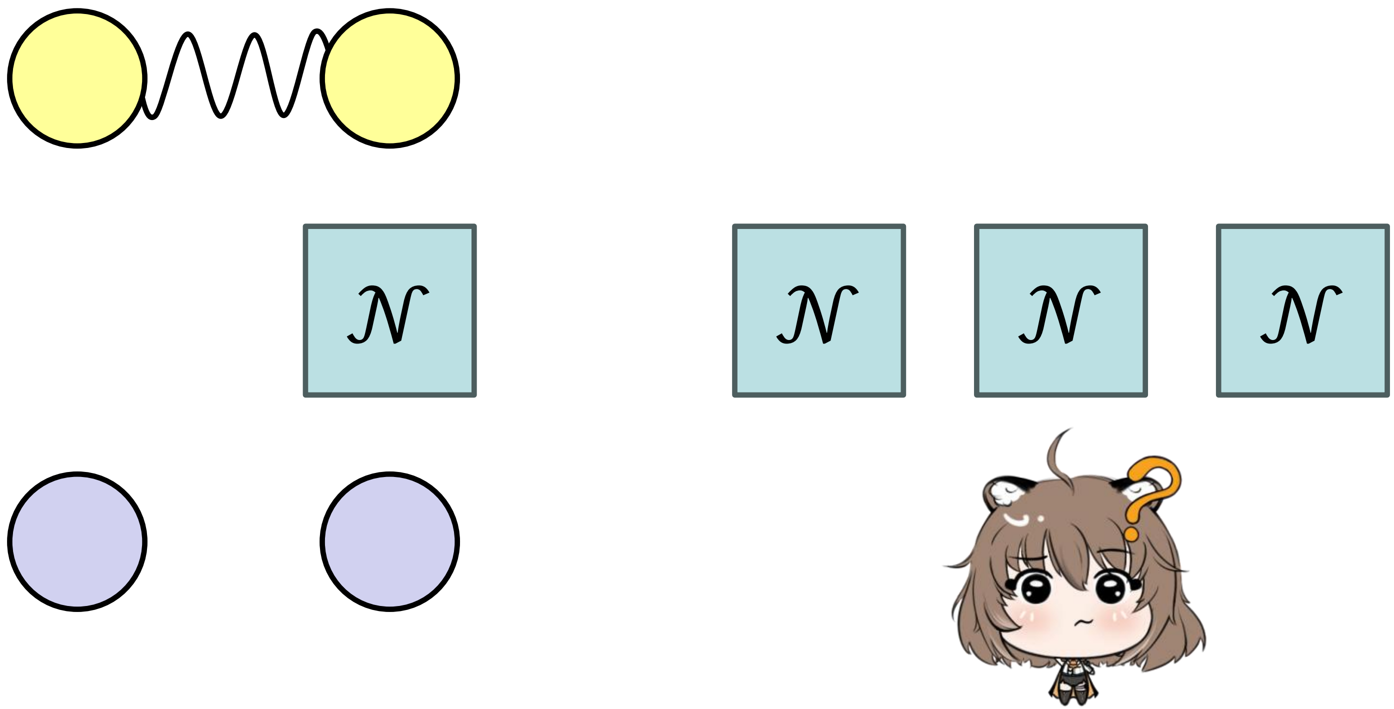

EB channels

Holevo form: measure-and-prepare channel

$$\mathcal{N}(\rho) = \sum_k \sigma_k \operatorname{tr}(F_k \rho)$$

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Choi state $\mathcal{J}_{AB}^{(\mathcal{N})}$ is separable

$$\mathcal{J}_{AB}^{(\mathcal{N})} = \mathrm{id}_A \otimes \mathcal{N}_{A\to B}(\Phi_{AA})$$

A channel is a quantum memory resource if it is non-entanglement-breaking

Main result

There exists compatible EB channels such that any broadcasting realization must generate entanglement.

$$\boxed{\mathcal{N}_{A\to B}}$$

$$\boxed{\mathcal{N}_{A\to C}}$$

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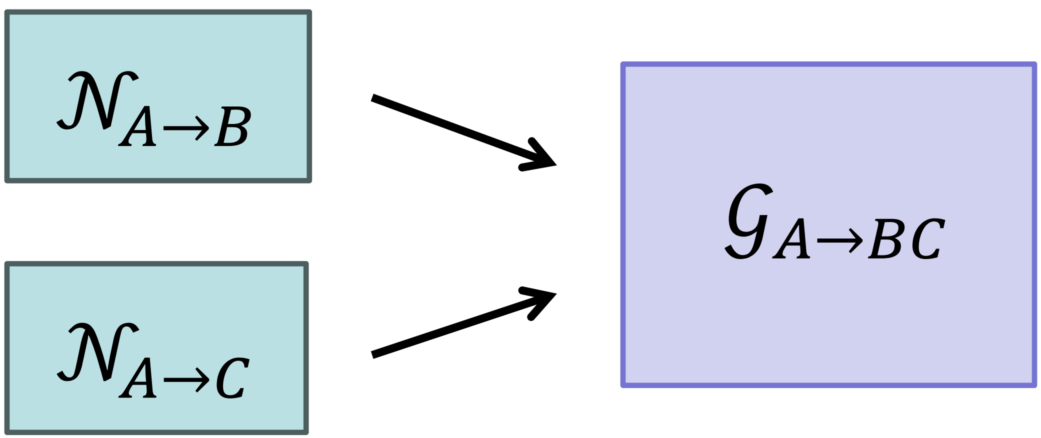

$$\mathrm{Tr}_C \circ \mathcal{G} = \mathcal{N}_{A\to B}$$

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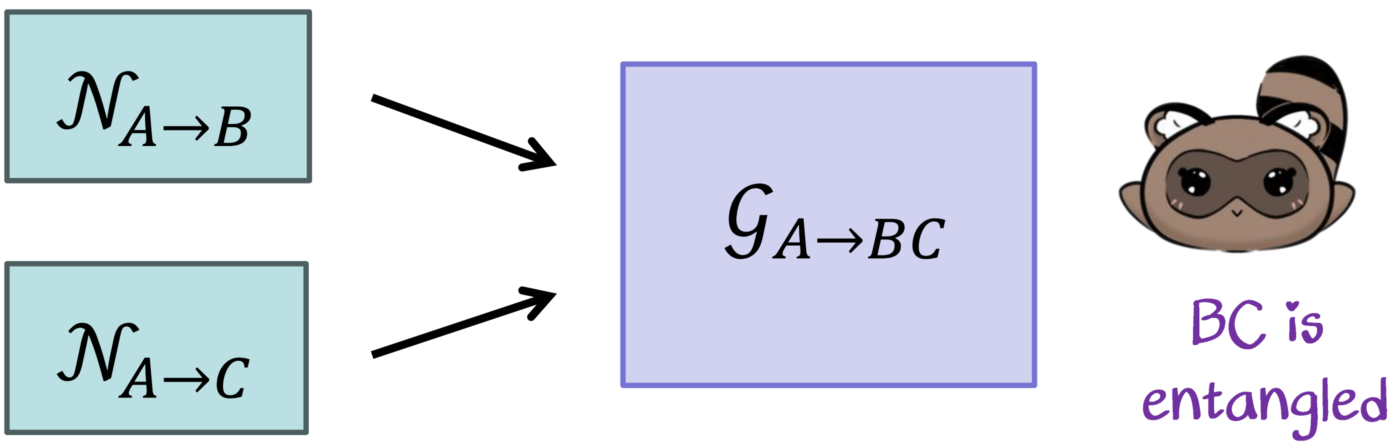
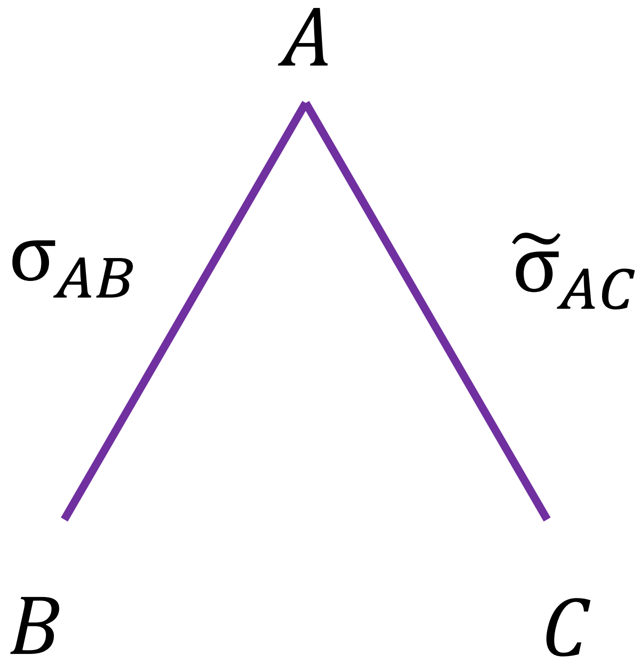


Entanglement transitivity

For all ρ_{ABC} with marginals $\rho_{AB} = \sigma_{AB}$ and $\rho_{AC} = \tilde{\sigma}_{AC}$, marginal ρ_{BC} is entangled.



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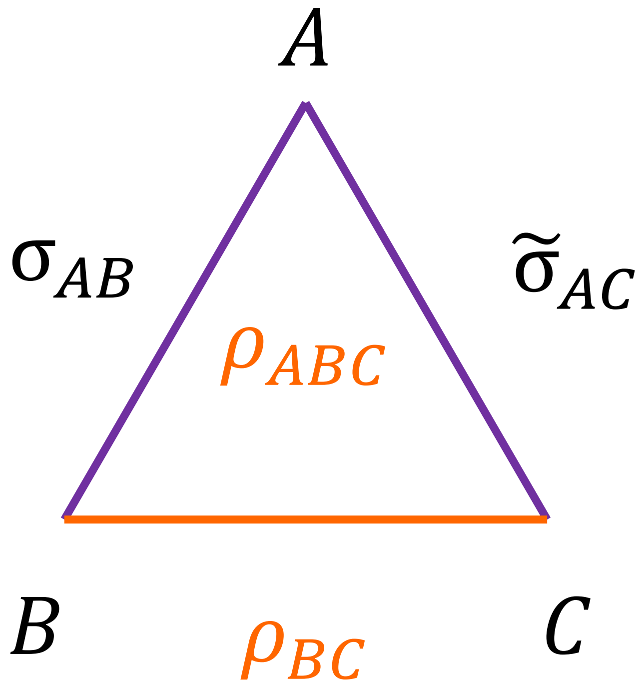
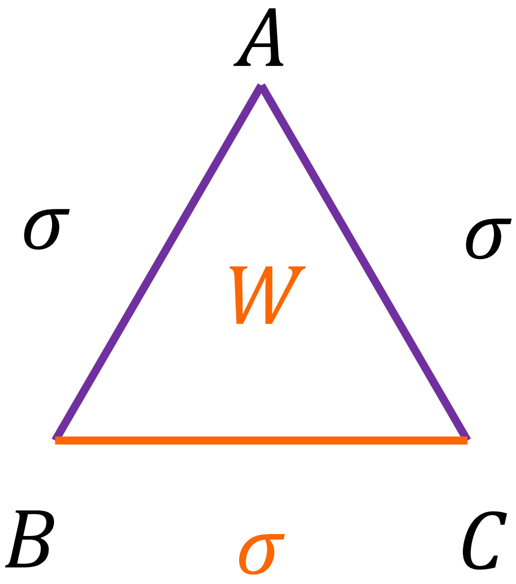


Example: W-state

$$\sigma_{AB} = \sigma_{AC} = \frac{2}{3}|\Psi^+\rangle\langle\Psi^+| + \frac{1}{3}|00\rangle\langle 00|$$

$$\Rightarrow |W\rangle = \frac{1}{\sqrt{3}}(|100\rangle + |010\rangle + |001\rangle)$$



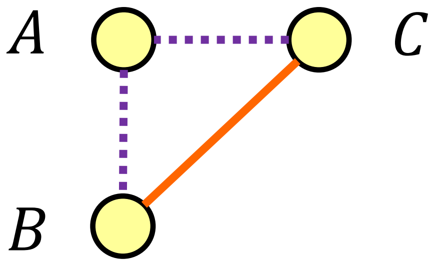
Certifying transitivity

When $\lambda^* < 0$, then ρ_{BC} is always entangled

$$
\begin{aligned}
\max_{\rho_{ABC}} \lambda &=: \lambda^* \\
\text{s.t.} \quad \rho_{ABC} &\geq 0 \\
\rho_{AB} &= \sigma_{AB} \\
\rho_{AC} &= \tilde{\sigma}_{AC} \\
\rho_{BC}^{T_B} &\geq \lambda \mathbb{I}
\end{aligned}
$$

Only separable marginals

A set of separable marginal states may also exhibit (meta)transitivity



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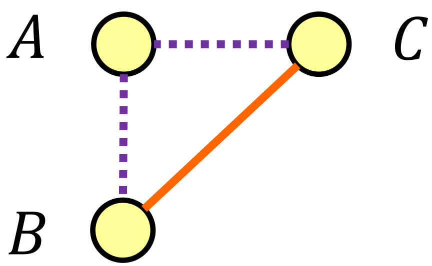

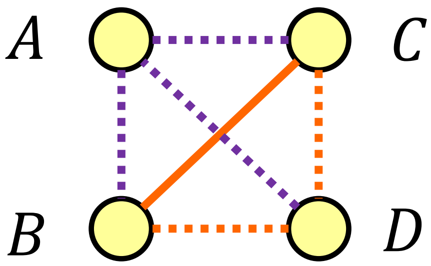

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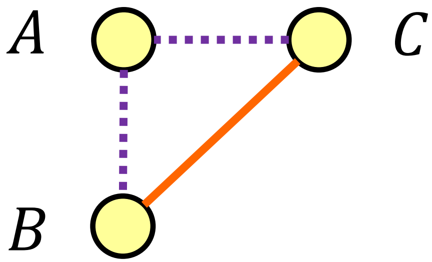
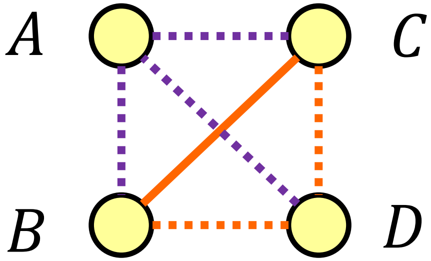
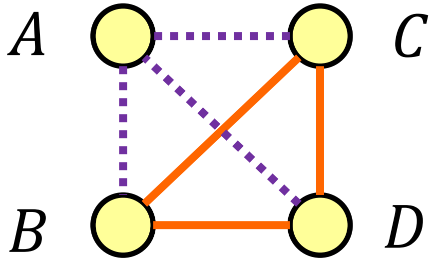






A specific example

$$|\psi_1\rangle_{ABC} = \sqrt{\tfrac{3}{20}}(|000\rangle + |111\rangle)_{ABC} + \sqrt{\tfrac{5}{20}}(|001\rangle + |110\rangle)_{ABC}$$
$$+ \sqrt{\tfrac{2}{20}}(|010\rangle + |101\rangle)_{ABC};$$
$$|\psi_2\rangle_{ABC} = \sqrt{\tfrac{5+\sqrt{15}}{40}}(|000\rangle + |001\rangle - |110\rangle - |111\rangle)_{ABC}$$
$$+ \sqrt{\tfrac{5-\sqrt{15}}{40}}(|010\rangle + |011\rangle - |100\rangle - |101\rangle)_{ABC}.$$

The reduced states ρ_{AB}, ρ_{AC} of $\rho_{ABC} = \frac{1}{2}(|\psi_1\rangle\langle\psi_1| + |\psi_2\rangle\langle\psi_2|)$ have $\rho_A = \frac{1}{2}\mathbb{I}_2$ and exhibit transitivity in BC

Broadcasting channel

When $\rho_A = \frac{1}{d}\mathbb{I}_d$, global state is Choi state of a broadcasting channel

$$
\begin{tikzcd}
& A \arrow[dl, "\mathcal{N}_{A\to B}"'] \arrow[dr, "\widetilde{\mathcal{N}}_{A\to C}"] & \\
B & & C
\end{tikzcd}
$$

$$\mathcal{J}^{\mathcal{G}}_{ABC} = (\mathrm{id}_A \otimes \mathcal{G}_{A\to BC})(\Phi_{AA})$$

Broadcast compatibility

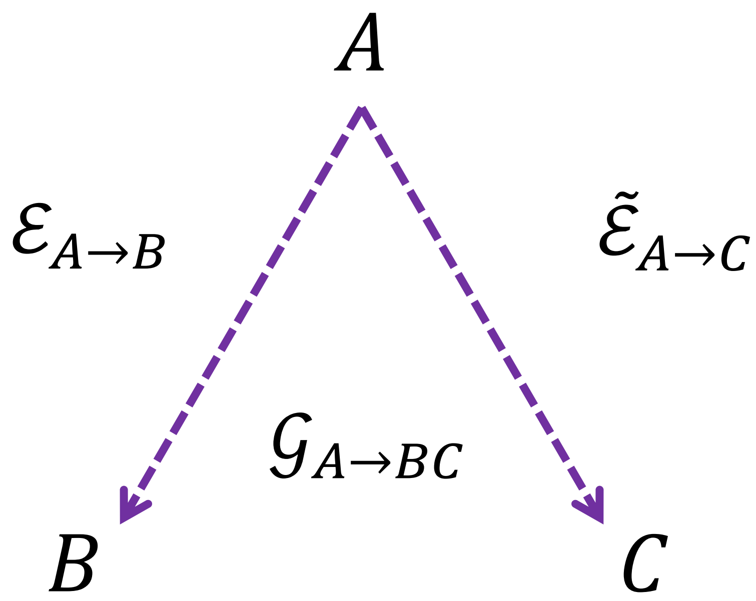
Pair $\mathcal{N}_{A\to B}, \widetilde{\mathcal{N}}_{A\to C}$ are broadcast-compatible if there is $\mathcal{G}_{A\to BC}$ such that

$$\mathrm{tr}_C \circ \mathcal{G}_{A\to BC} = \mathcal{N}_{A\to B},$$
$$\mathrm{tr}_B \circ \mathcal{G}_{A\to BC} = \widetilde{\mathcal{N}}_{A\to C}.$$

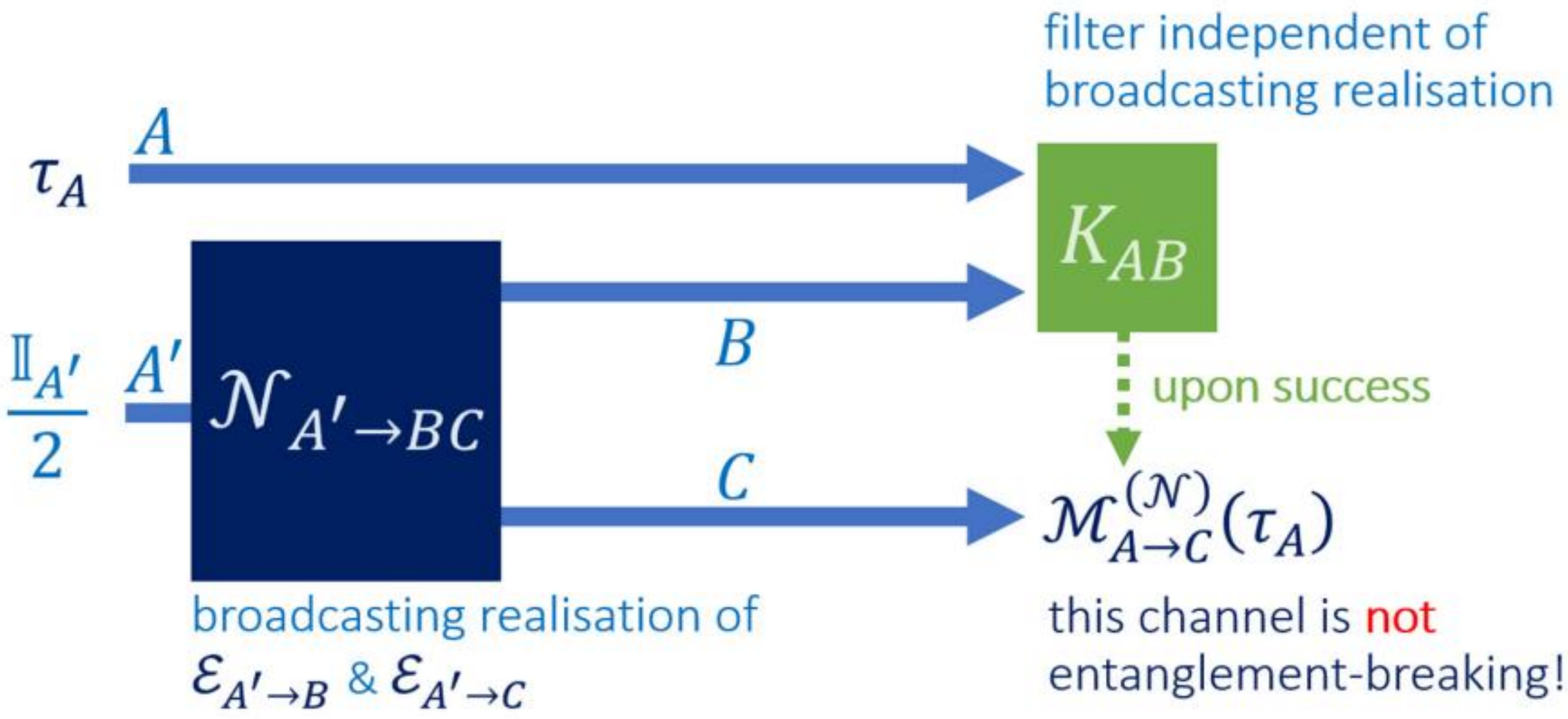
Broadcasting realizations $\mathcal{G}_{A\to BC}$ are not generally unique

Generates entanglement

For EB channels $\mathcal{E}_{A\to B}, \tilde{\mathcal{E}}_{A\to C}$ with transitivity,
$\mathcal{G}_{A\to BC}(\frac{1}{2}\mathbb{I}_2)$ is always entangled for all $\mathcal{G}_{A\to BC}$

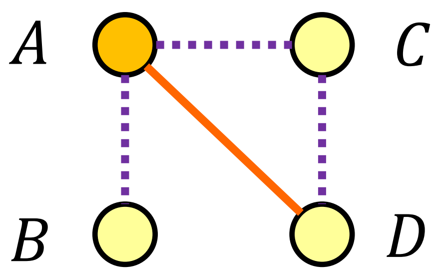


Superactivation of QM



Beyond three qubits

Four-qubit state with $\rho_A = \frac{1}{2}\mathbb{I}_2$ and AB, AC, CD exhibit meta-transitivity in AD



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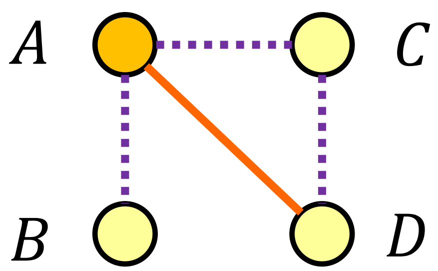
With many copies of $\mathcal{G}_{A\to BCD}$, verify correct reduced state in CD



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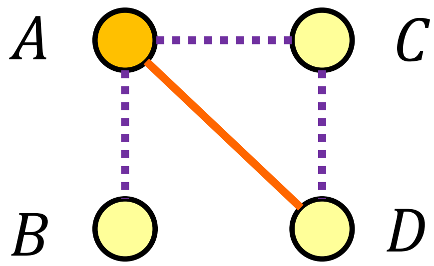
If success: $\mathrm{tr}_{BC} \circ \mathcal{G}_{A\to BCD} = \mathcal{N}_{A\to D}$ is non-EB



Take home message

Using compatibility and EB property, we can super-activate quantum memory.

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Applied meta-transitivity of states to super-activation of channels

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Future directions

Compatibility of resource-breaking channels can be resource-generating

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<u>Future directions</u>

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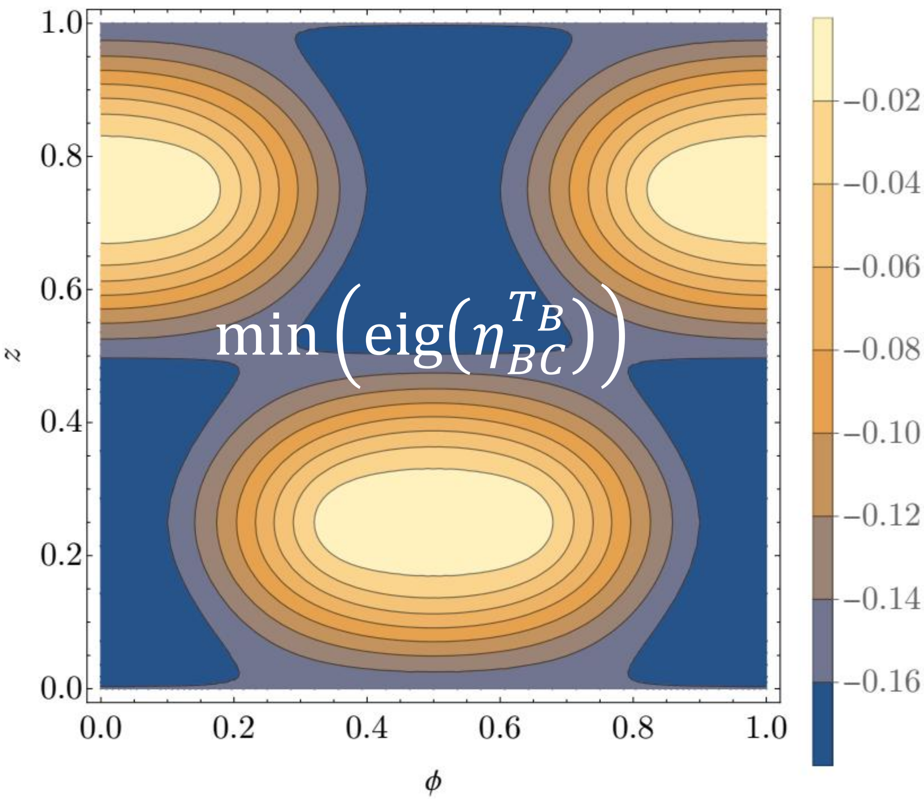
Other applications of meta-transitivity

Generic pure input



$$\mathcal{G}_{A\to BC}\left(\Psi_{z,\phi}\right) =: \eta_{BC}$$

Broadcast channel $\mathcal{G}_{A\to BC}$ generates entanglement in BC for random $\left|\psi_{z,\phi}\right\rangle$

$$\left|\psi_{z,\phi}\right\rangle = \cos\pi z\,|0\rangle + e^{i\phi}\sin\pi z\,|1\rangle$$